technische universiteit eindhoven

MathVisionTools

A high level Mathematica library for image processing

Prof. Bart M. ter Haar Romeny, PhD
Dipl.ing. Markus van Almsick
Eindhoven University of Technology
Department of Biomedical Engineering
B.M.terHaarRomeny@tue.nl

http://www.tue.nl/

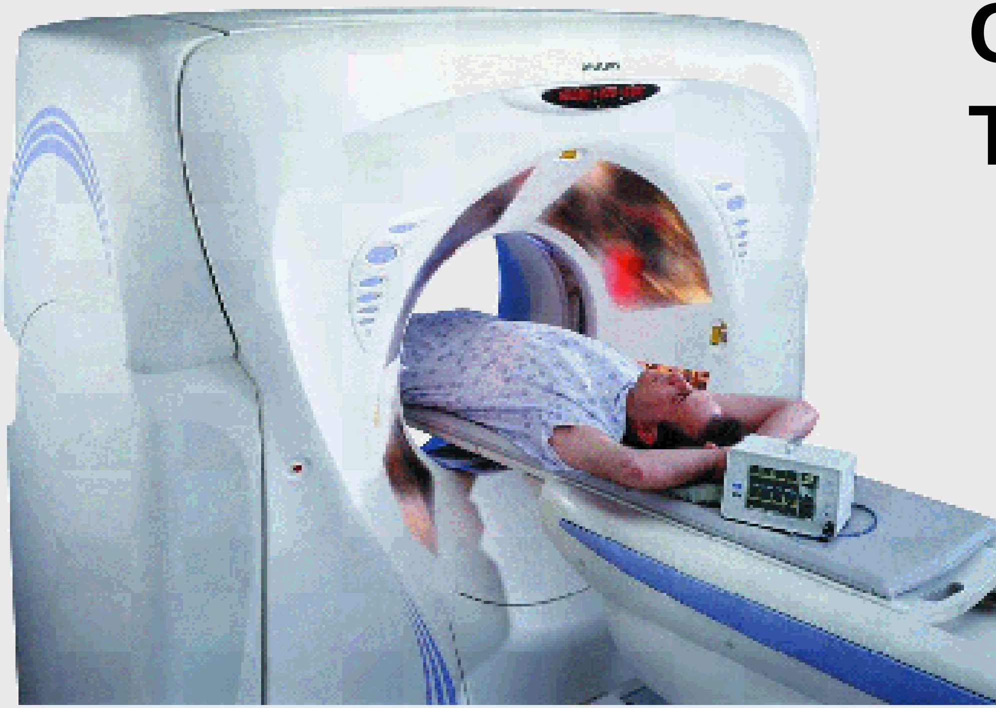
Medical imaging is big business:

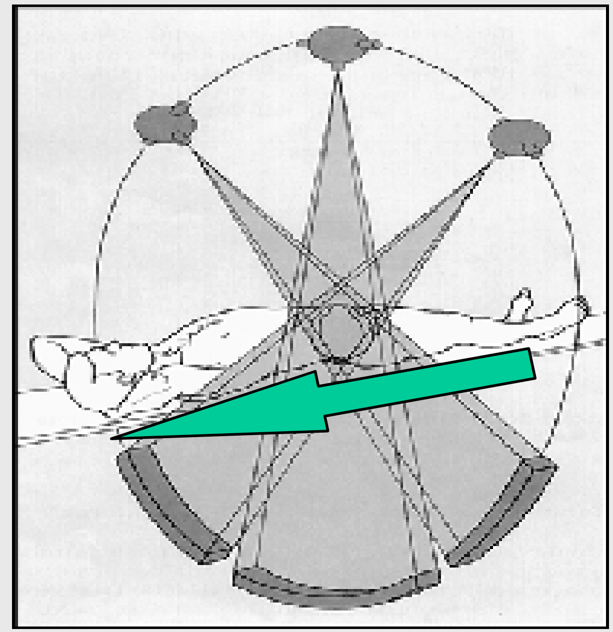
- One third of hospital’s equipment is for medical imaging
- 80% of all diagnoses are done on images
- Typical 800 bed hospital produces 10 Terabyte/year
- Sector grows steadily by 10% per year
- GE, Philips, Siemens: billions of dollars markets

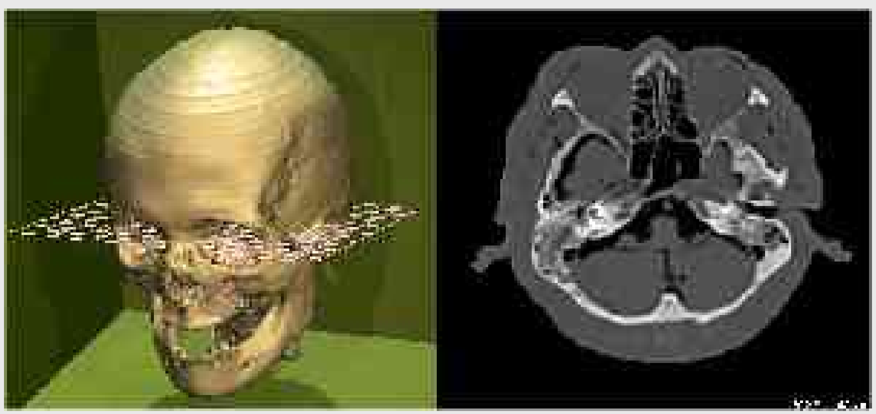
Medical images are big:

- CT and MRI scan: typically 512 x 512, 800 to 2000 slices, 16 bit (400 MB – 1 GB)
- Digital X-Ray, mammogram: 3000 x 2500 pixels, 16 bit (15 MB)
- Ultrasound: 256 x 256, 20 frames per second, 8 bit (80 MB/min)

Computer Tomography



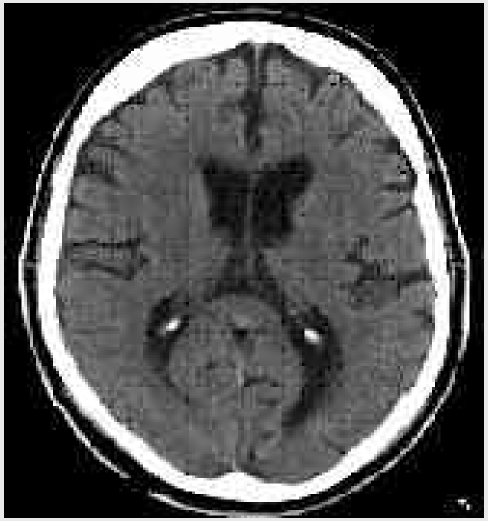


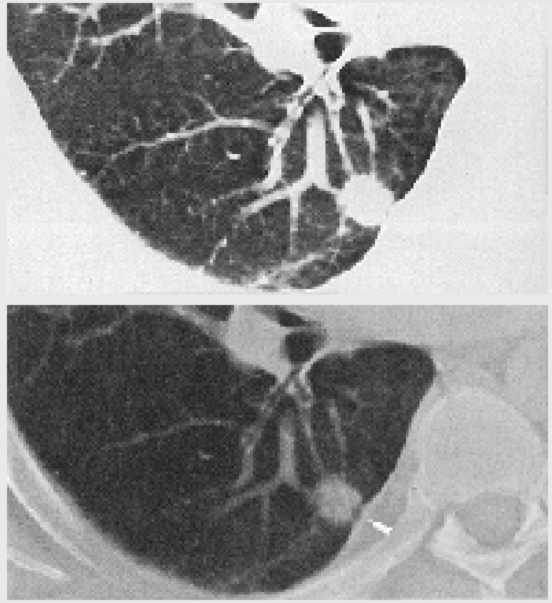


Multi-slice CT: 4-64 slices per rotation (0.5 sec).

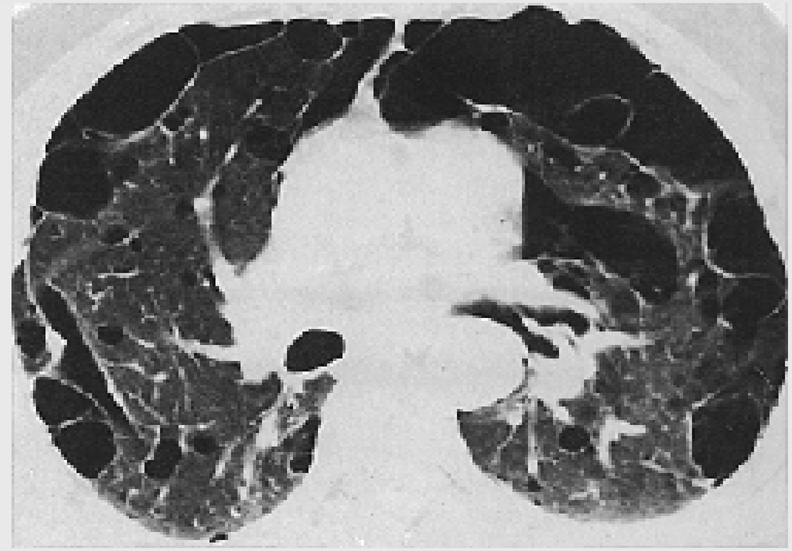
Full body trauma scan: 21 sec

Applications

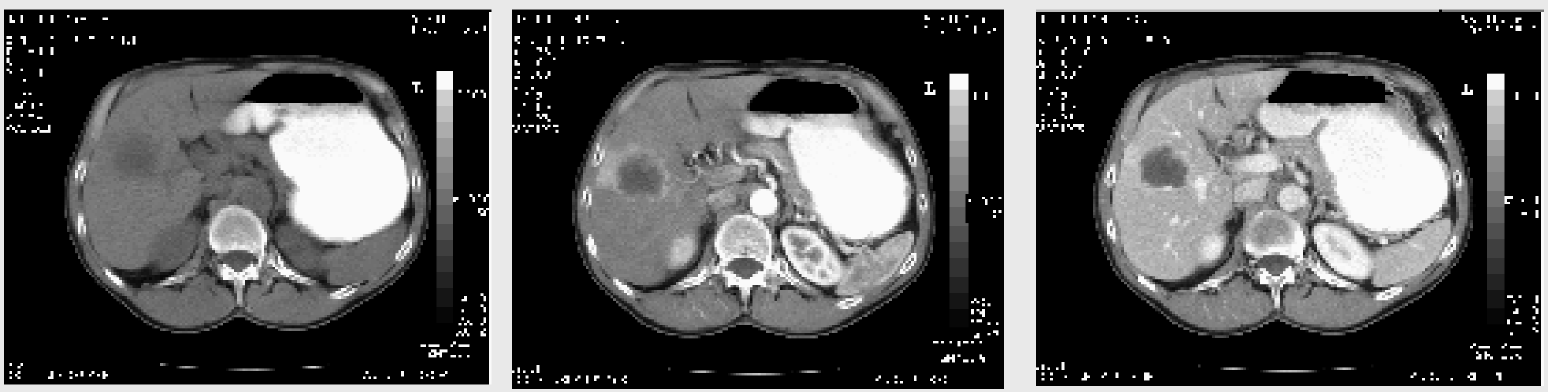




peripheral bronchial carcinoma = long tumour

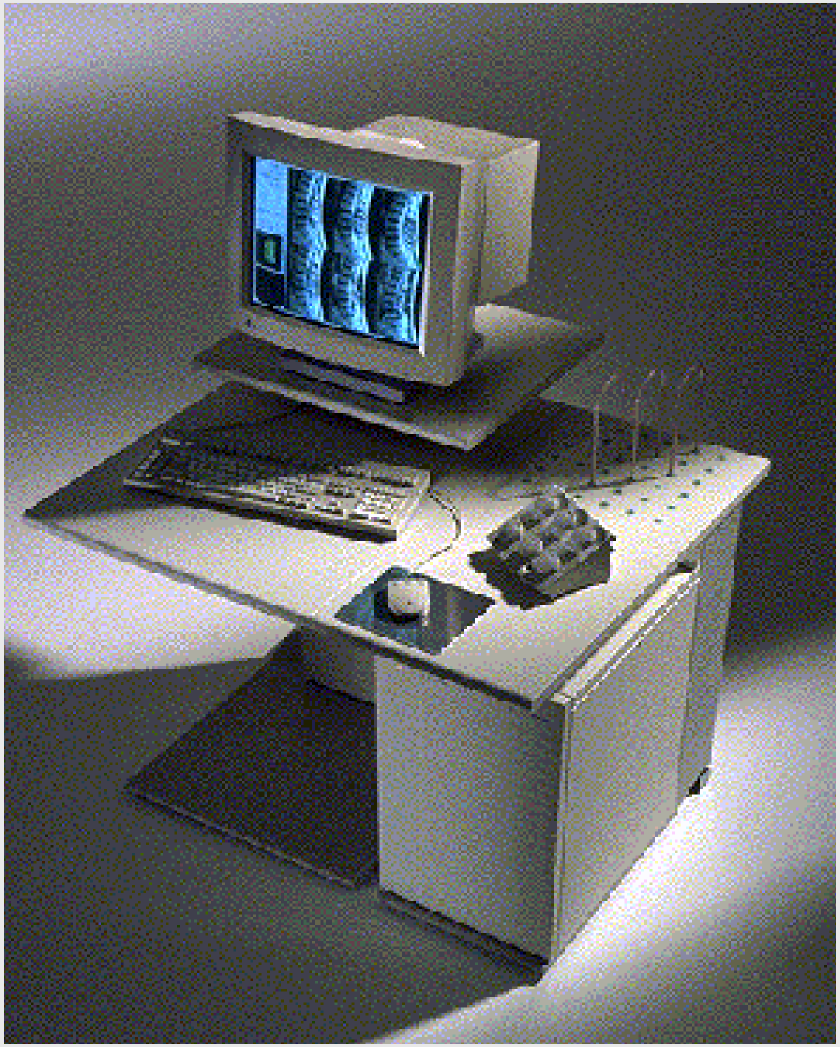


emfyseme



multi-phase study of the liver

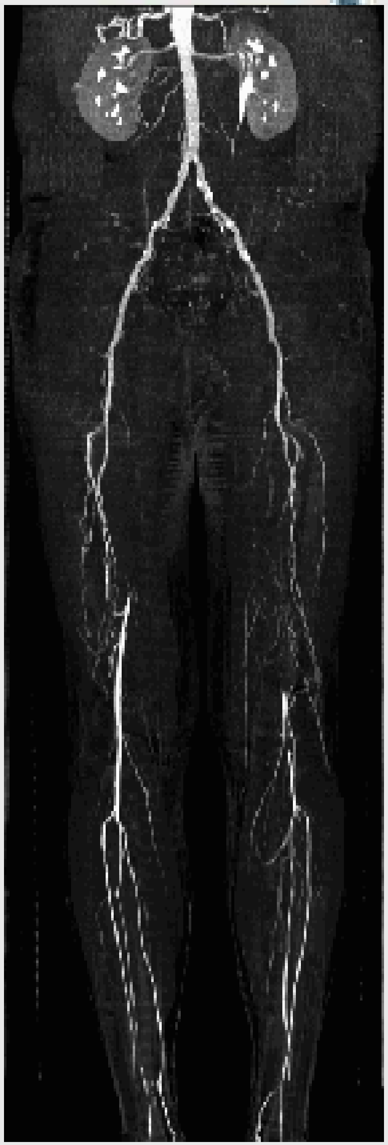
Magnetic Resonance Imaging



Medical workstations quickly become the super-assistants of radiologists and surgeons

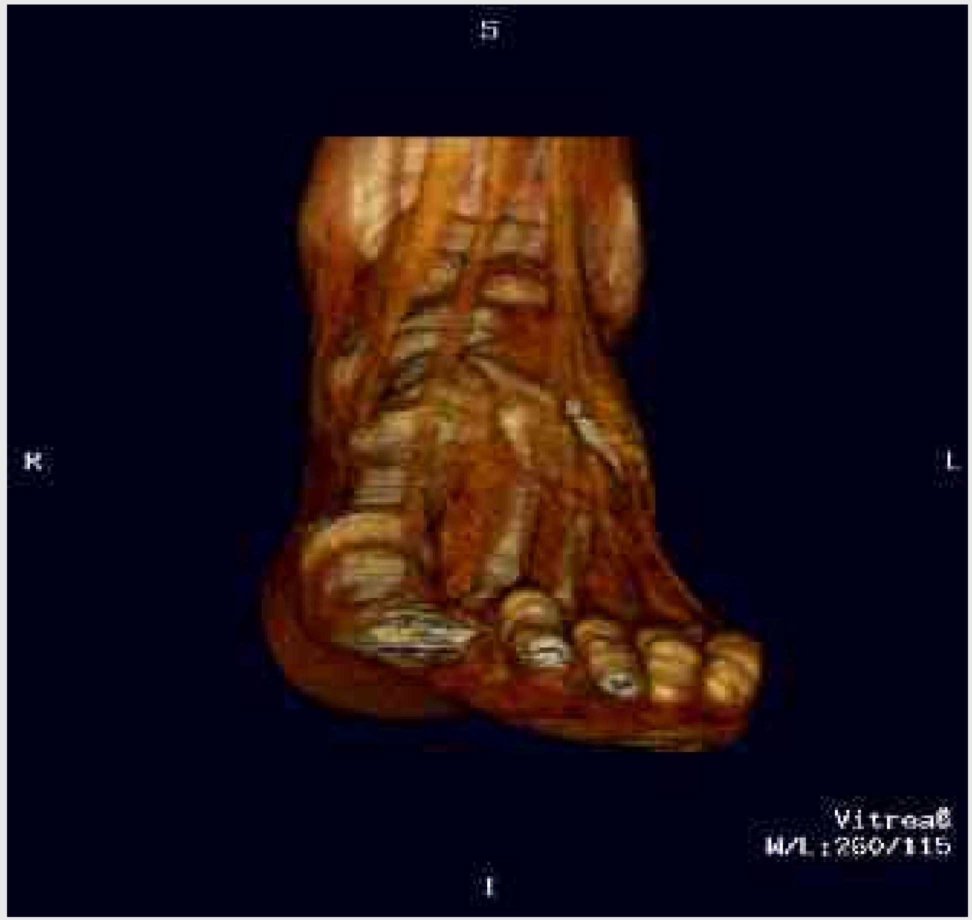
- 3D visualization
- Computer aided diagnosis
- Surgery planning
- Virtual endoscopy
- Tissue classification
- Interactive analysis
- etc.

The lightbox has disappeared, now hundreds of workstations



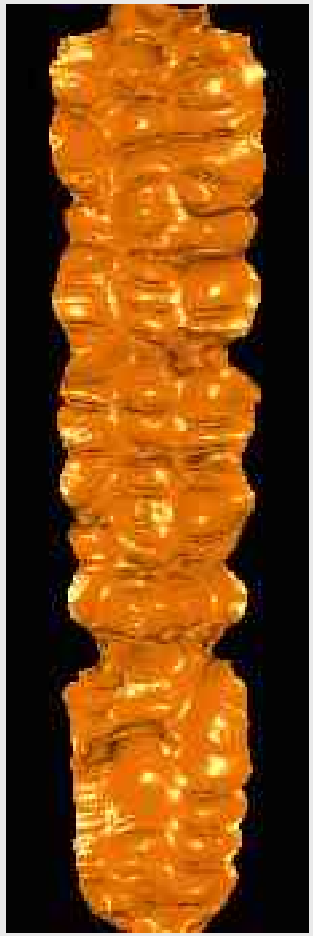
Maximum Intensity Projection

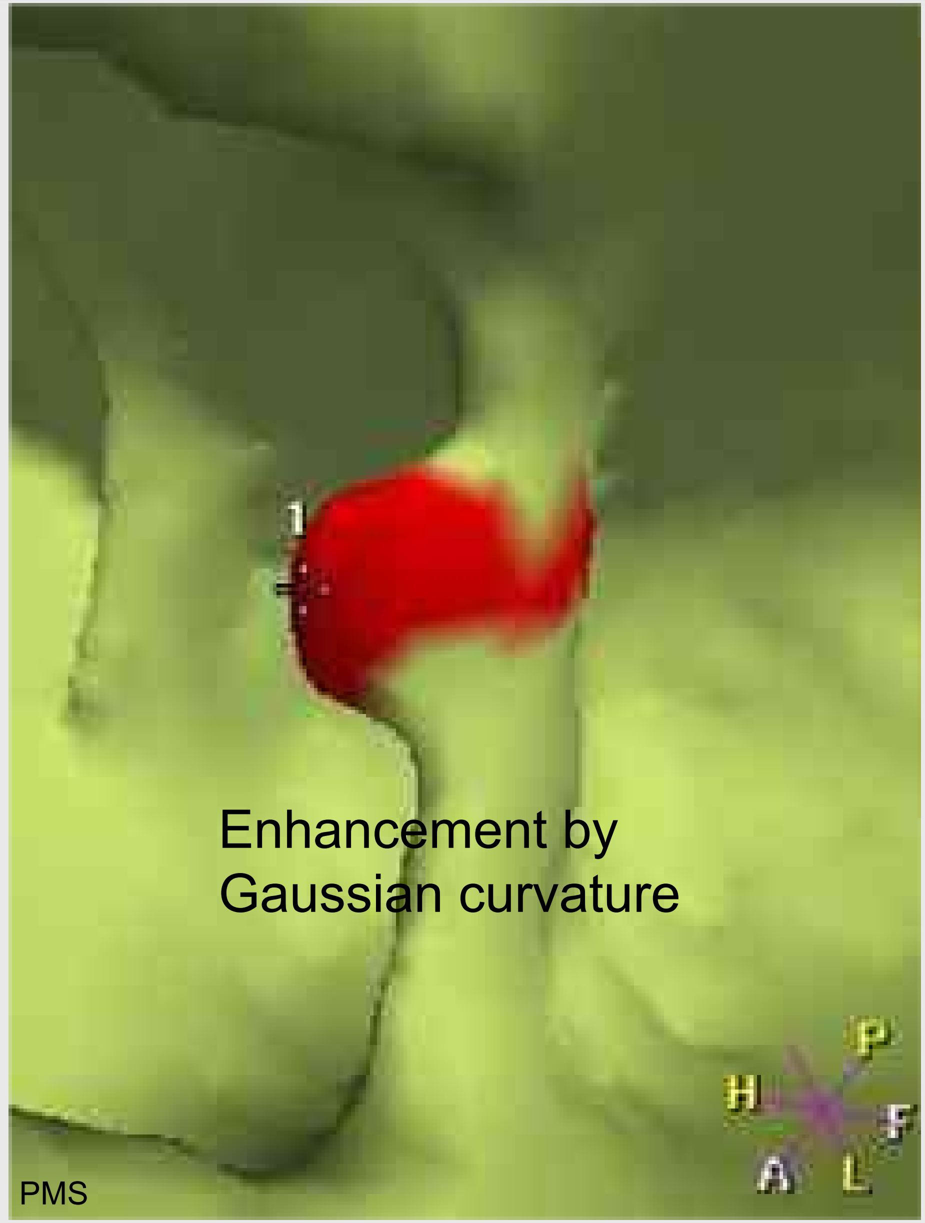
De overwhelming amount of data calls for condensed presentation and analysis:
Strong demands on front-end visualizations




ZIB
amira

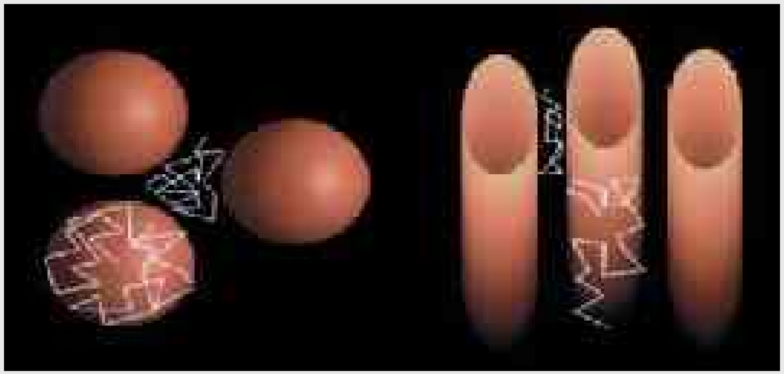
Advanced volume visualization



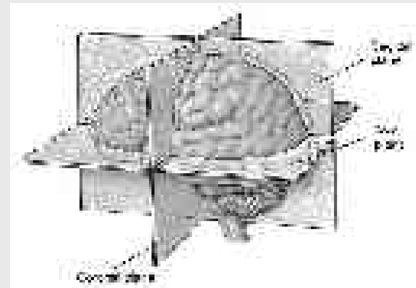


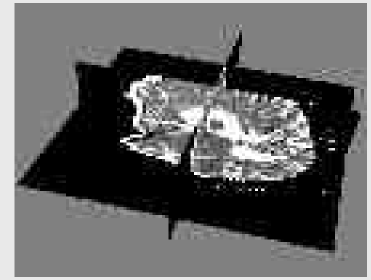
Virtual endoscopy

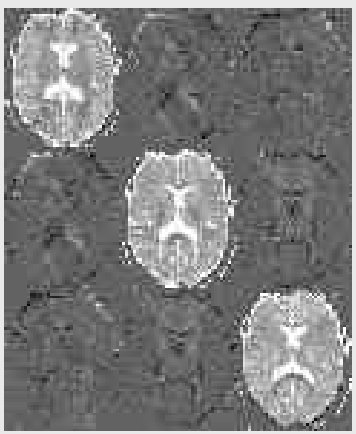
Diffusion Tensor Imaging - visualization



Brownian motion of water:
diffusion (sphere - ellipsoid)

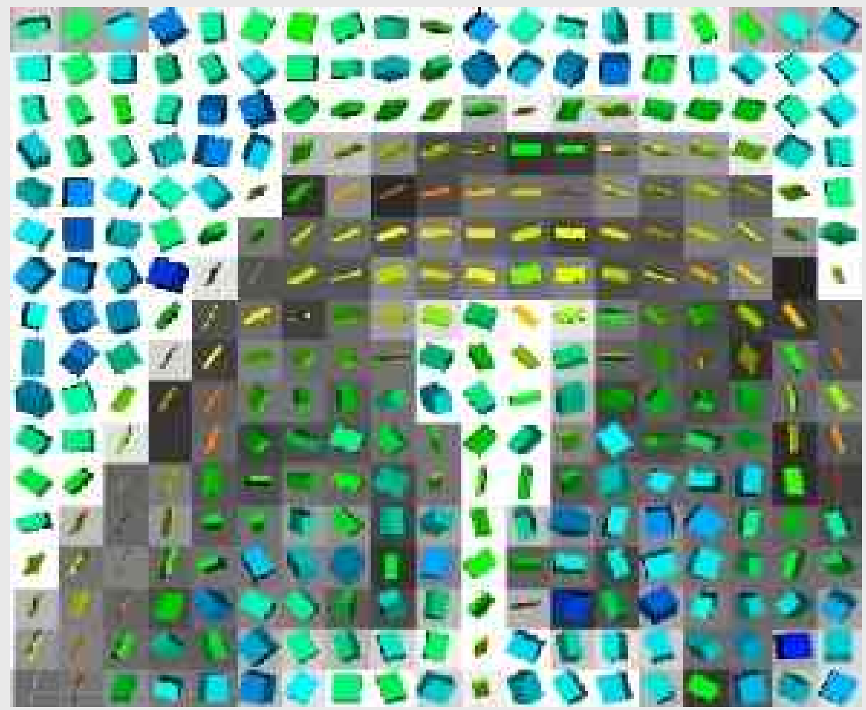


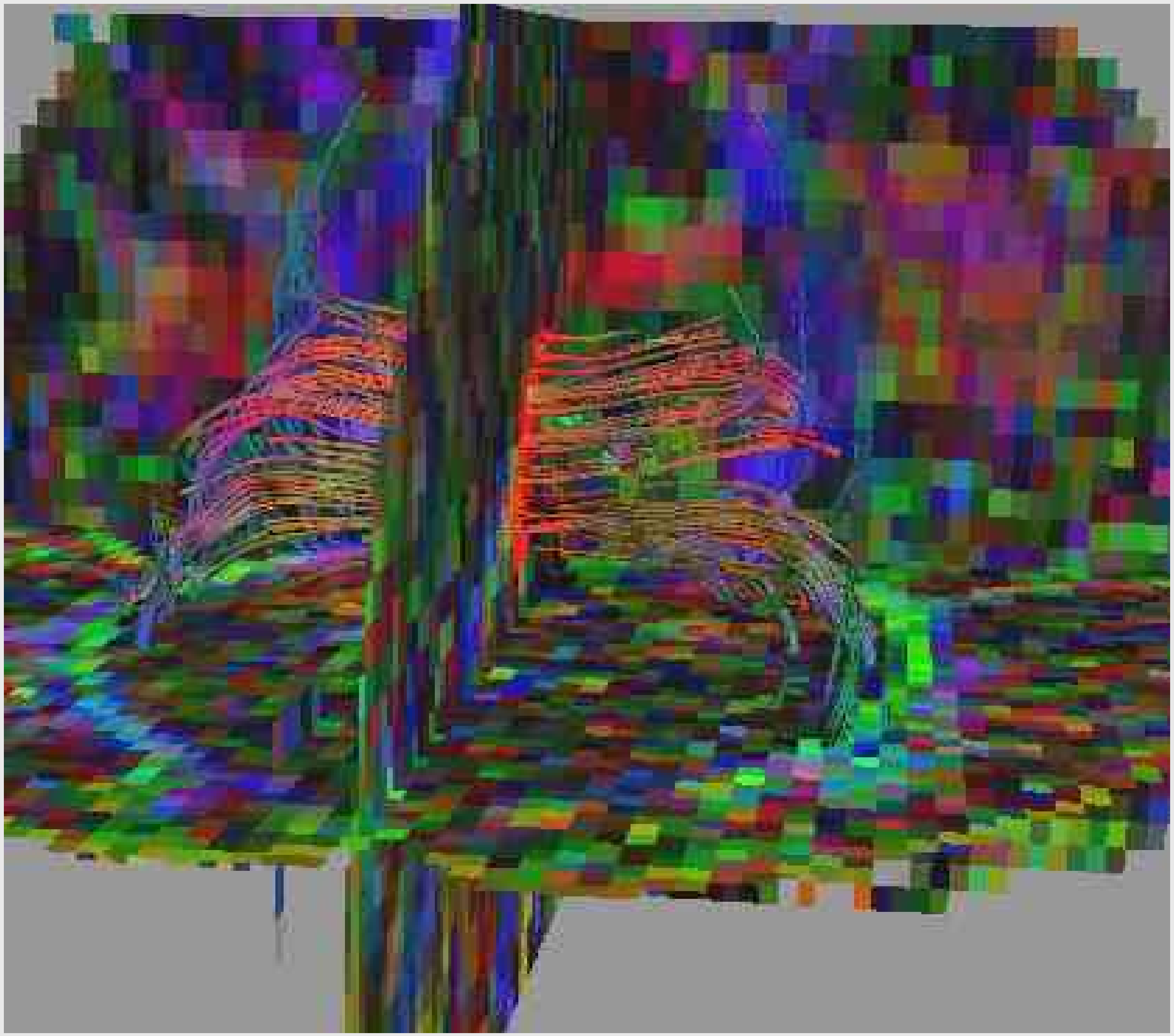




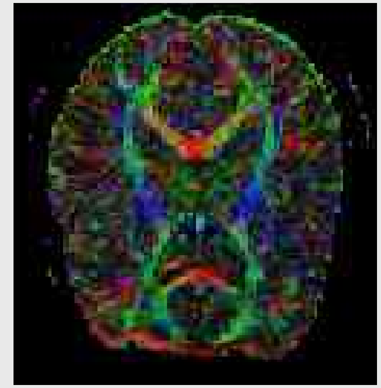
Tensor components

Eigenvectors





Berenschot – Vilanova – TUE/BMT



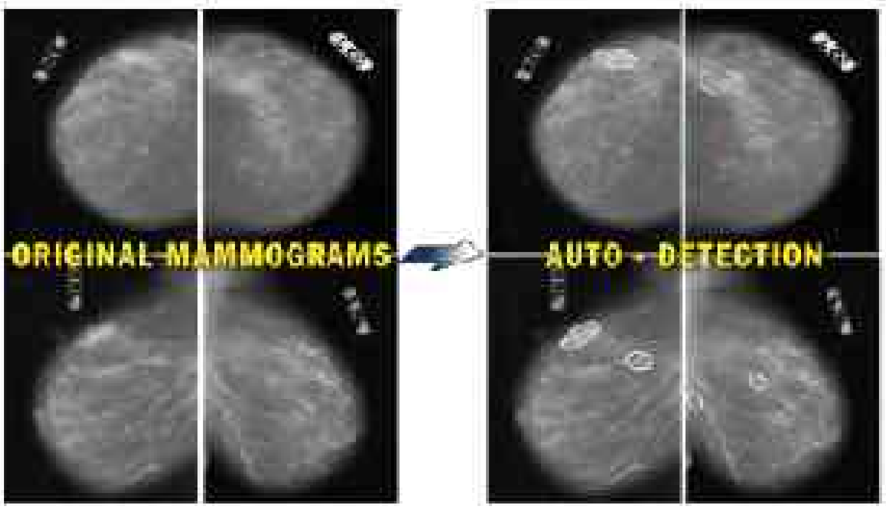
Color indicates orientation

Visual ToolKit

www.vtk.org

Computer Aided Diagnosis

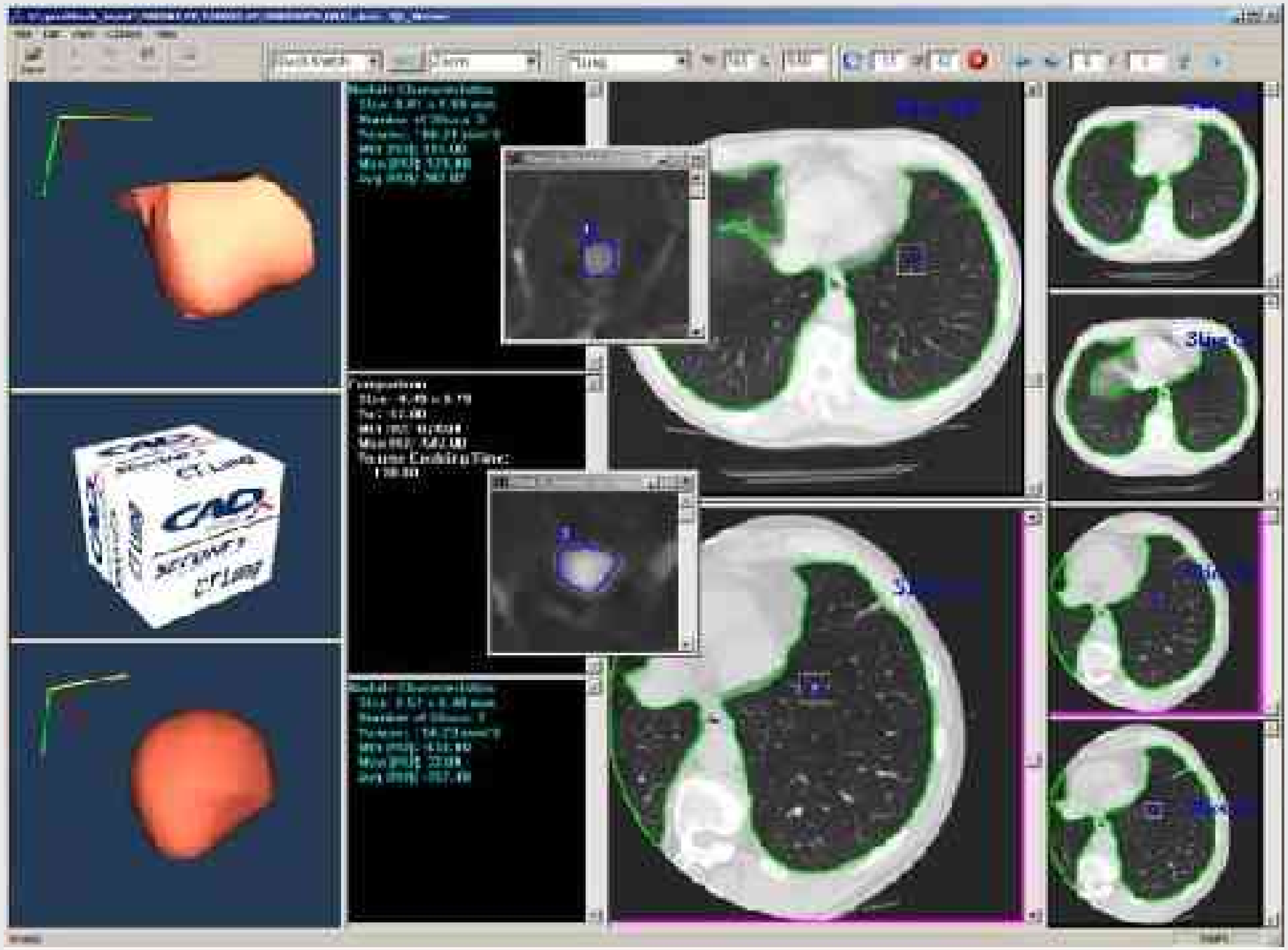
Mammography:



CADx

In a prospective study based on screening exams performed on almost 13,000 consecutive women over a one-year period, Ulissey and colleagues found that CAD increased breast cancer detection by 20% (*Radiology*, September 2001, Vol. 220:3, pp. 781-786).

CT lung pathology

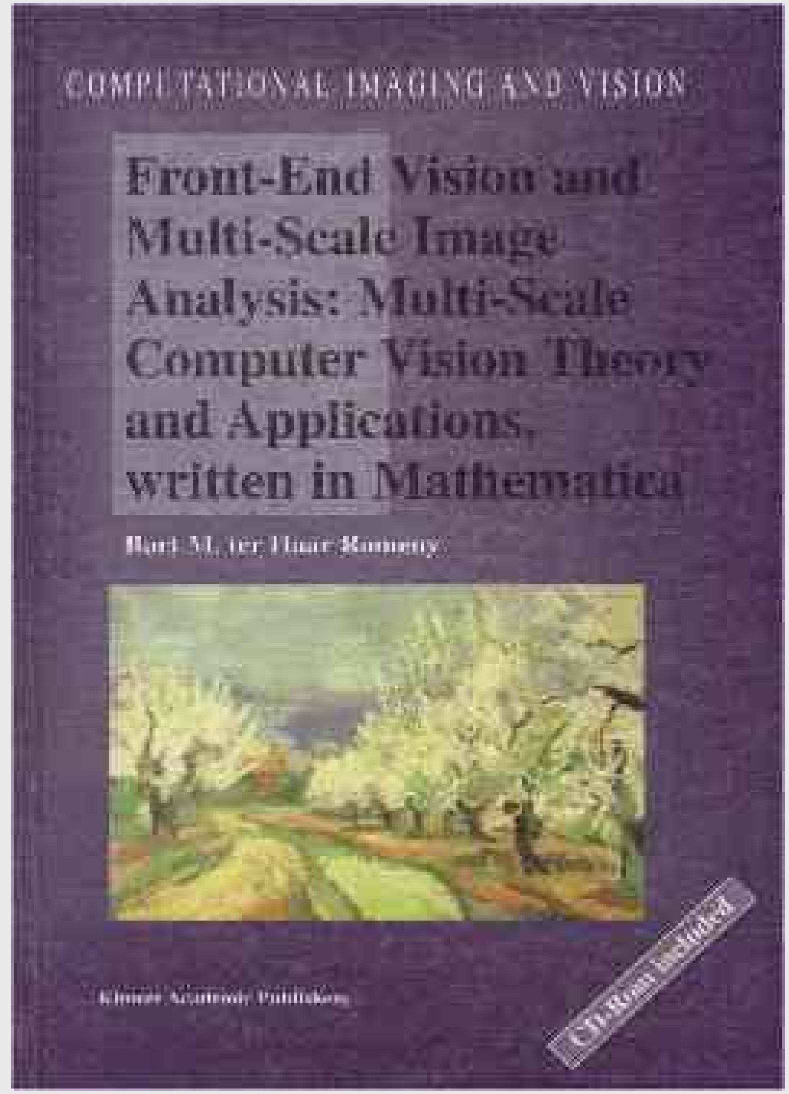


CADx

Companies:

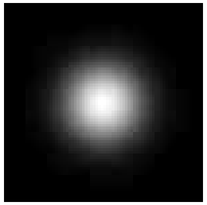
R2 Technologies
Deus Technologies
CADVision
iCAD
CadX
Philips
GE
Siemens
...

Multi-Scale Image Analysis

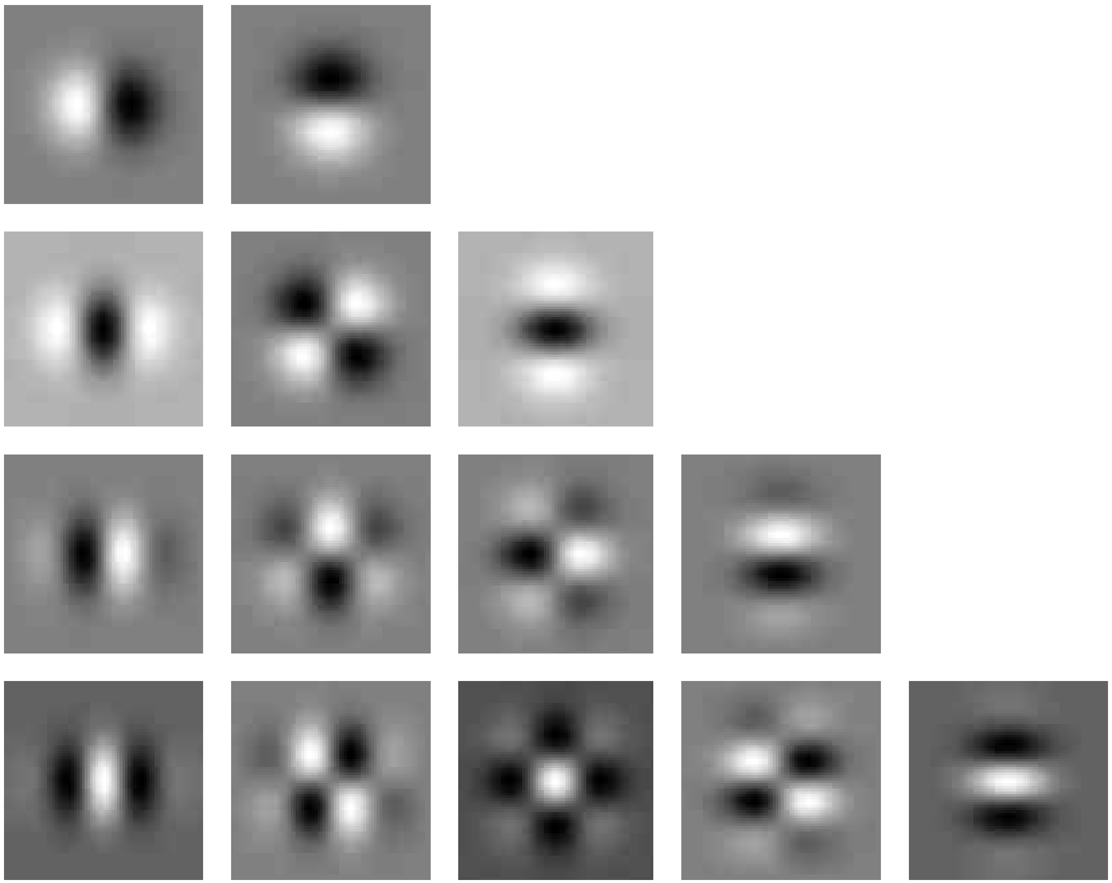


Biologically inspired computer vision

→ **<u>bio-mimicking</u>**



Gaussian derivative profiles up to 4th order

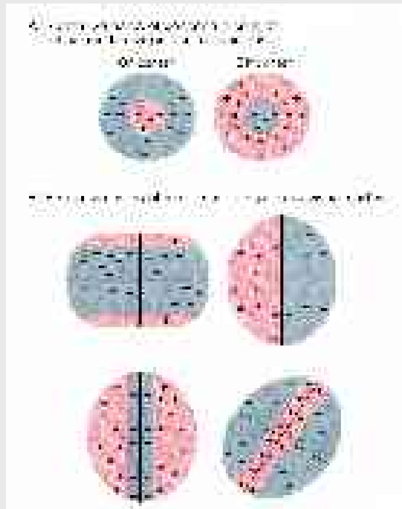


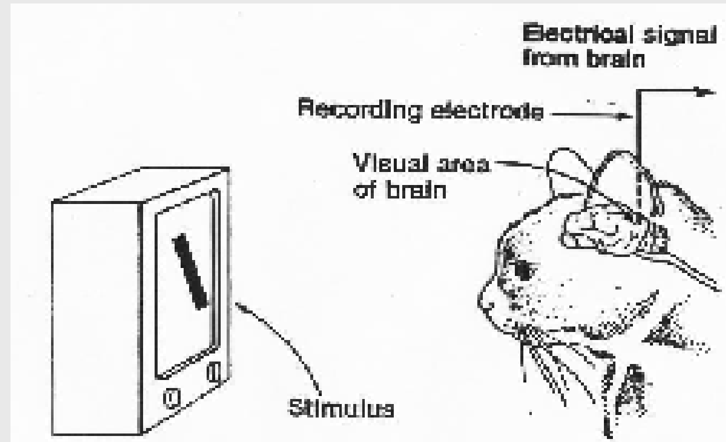
Observation, sampling
= convolution by aperture

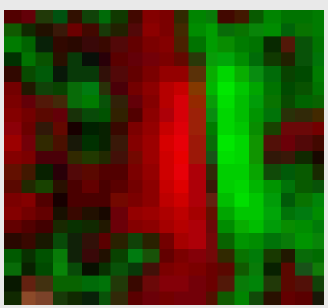
$$\frac{\partial L}{\partial x}(L \otimes G) = L \otimes \frac{\partial}{\partial x} G$$

Differentiation of discrete data is done by convolution with Gaussian derivative kernels

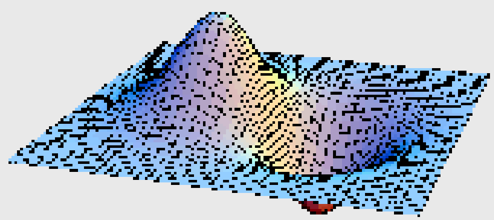
Many cells in the visual cortex function as derivative operators







Model:
several orders
Gaussian
derivatives



Receptive fields measure spatio-temporal structure

differential geometry

MathVisionTools

List of currently available functions:

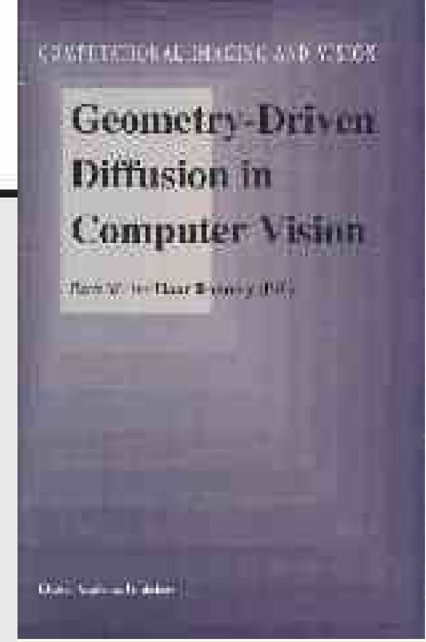
- Differential Geometry
    - Gaussian derivatives
    - for any order
    - for N dimensions
- Import / Export
    - any dialect DICOM
    - high field MRI
    - 3D ultrasound
    - 2-photon microscopy
- Orientation analysis
    - Polar Fourier Transform
    - 2D Hankel Transform

Soon available:

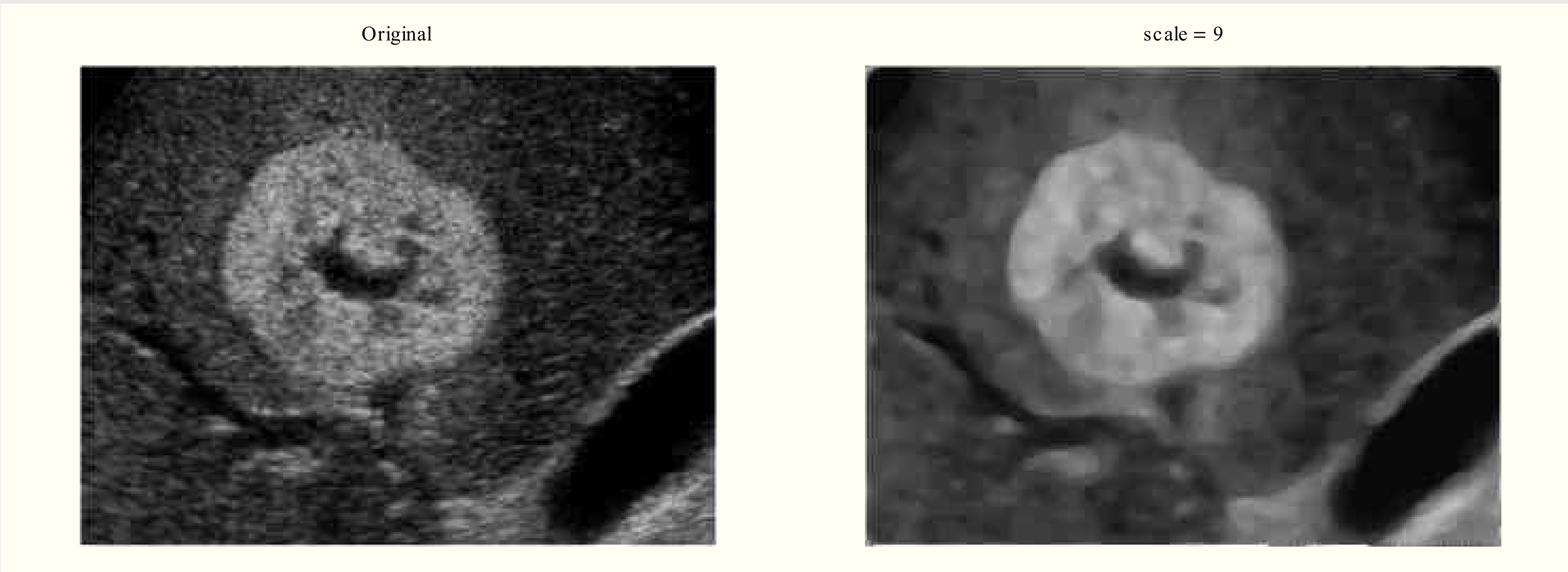
- MIP (perspective and orthogonal)
- Image registration in 2D and 3D
- Mutual entropy & correlation measure
- Lung nodule detection
- Catheter detection by 3D orientation bundle
- Tensor voting for perceptual grouping
- Dynamic shape Eigenmode analysis
- Automatic updating system

Scheduled:

- Nonlinear image registration
- PDE based edge preserving smoothing
- Motion from dense optic flow fields
- Active contours atlas mapping
- Snakes and levelsets
- Image retrieval (multi-scale)
- Multi-scale texture classification



Geometry-driven diffusion:
edge preserving smoothing



A *conductivity coefficient* (c) is introduced in the diffusion equation:

$$\frac{\partial L}{\partial s} = \vec{\nabla}.c\,\vec{\nabla} L \qquad c = c\left(L, \frac{\partial L}{\partial x}, \frac{\partial^2 L}{\partial x^2}, \ldots\right)$$

It is a *divergence* of a *flow*. We also call $c\,\vec{\nabla} L$ the *flux function*. With c = 1 we have normal linear, isotropic diffusion: the divergence of the gradient flow is the Laplacian.

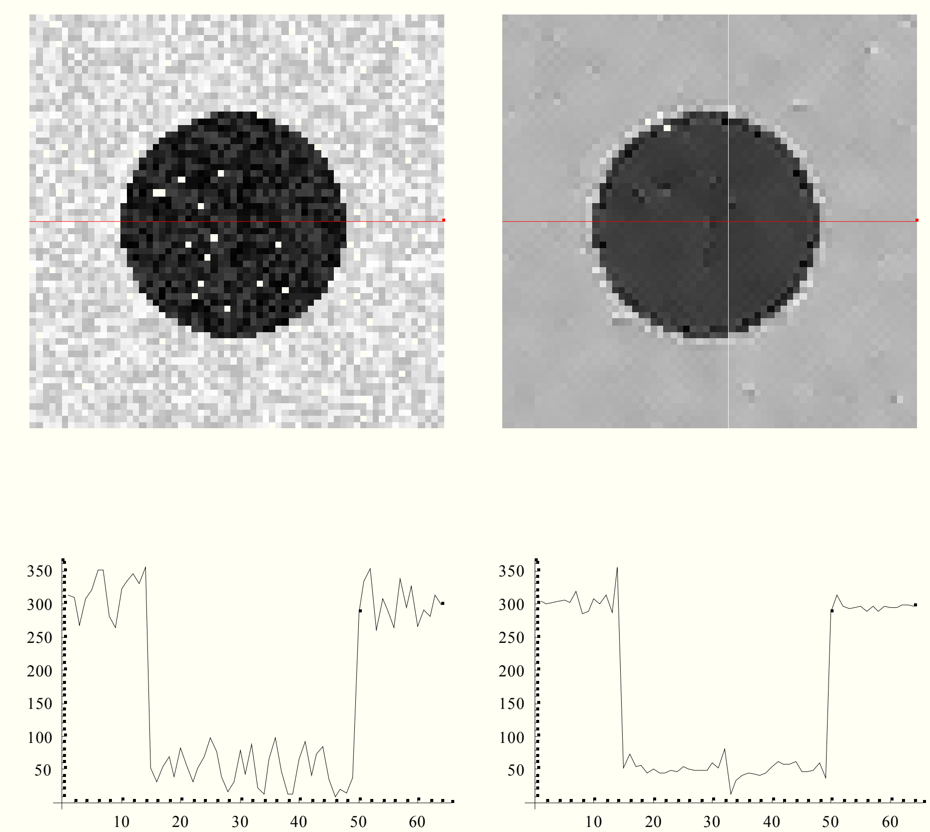
The Perona & Malik equation (1991): $c_1 = e^{-\frac{|\vec{\nabla} L|^2}{k^2}}$ $\quad \frac{\partial L}{\partial s} = \vec{\nabla}.c(|\vec{\nabla} L|)\vec{\nabla} L$

$$\frac{e^{-\frac{L_x^2+L_y^2}{k^2}}\left((k^2 - 2\,L_x^2)\,L_{xx} - 4\,L_x\,L_{xy}\,L_y + (k^2 - 2\,L_y^2)\,L_{yy}\right)}{k^2}$$

The solution is knot known analytically, so we have to rely on numerical methods, such as the forward Euler method.

$$\delta L = \delta s\,(\nabla . c \nabla L)$$

Test on a noisy test image:



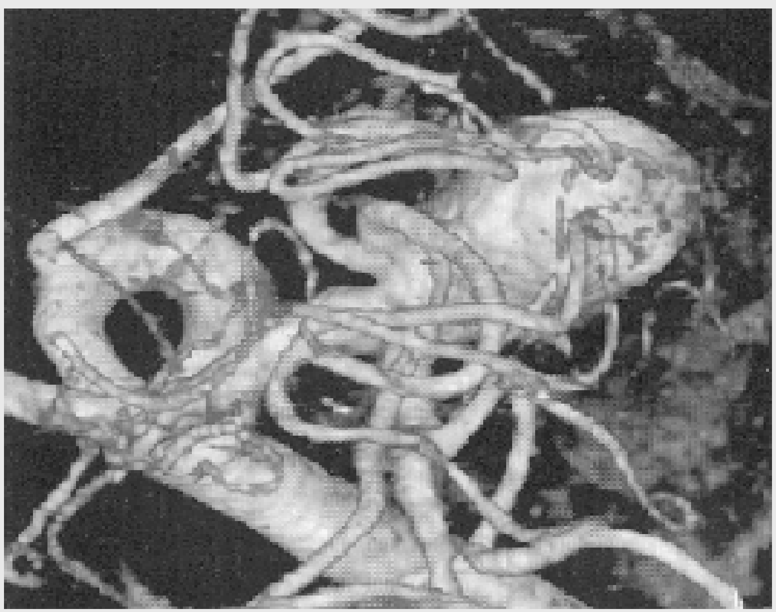
Note the preserved steepness of the edges with the strongly reduced noise.

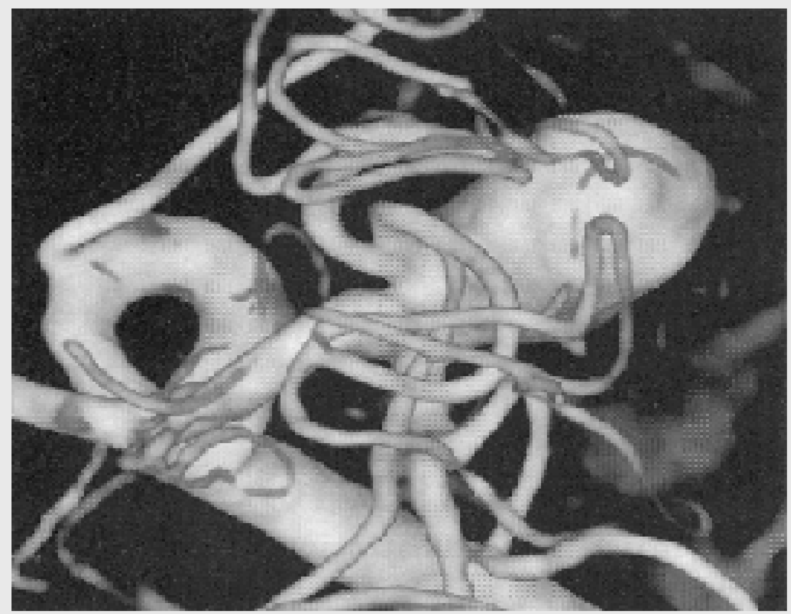
In 3D:

$$\frac{\partial L}{\partial t} = \vec{\nabla} \cdot e^{-\frac{\|\vec{\nabla} L\|^2}{k^2}} \vec{\nabla} L =$$

$$\frac{1}{k^2} e^{\frac{L_x^2 + L_y^2 + L_z^2}{k^2}} \left(k^2 (L_{xx} + L_{yy} + L_{zz}) - 2 (L_x^2 L_{xx} + L_y^2 L_{yy} + L_z^2 L_{zz}) \right.$$

$$\left. -4 (L_x L_y L_{xy} + L_x L_z L_{xz} + L_y L_z L_{yz}) \right)$$



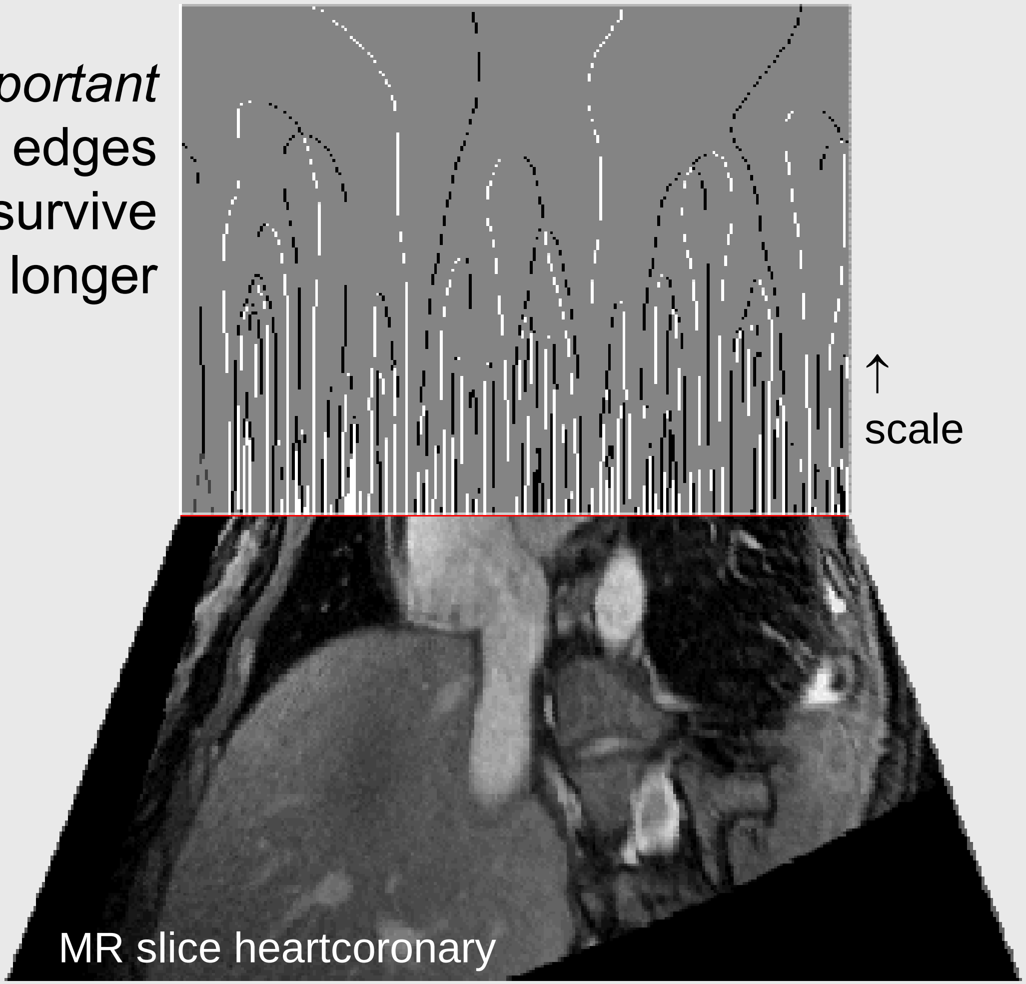


If conductivity is dependent
on the second order structure tensor:

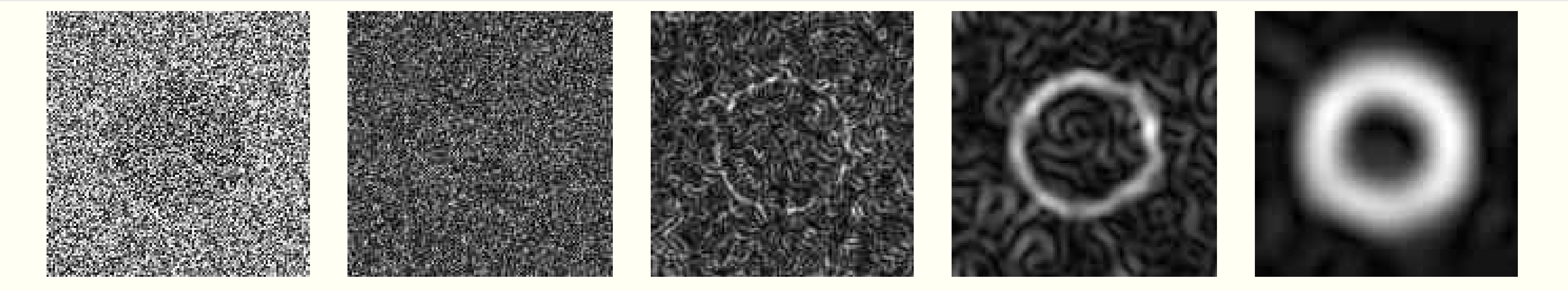
Coherence enhancing diffusion

J. Weickert, 2001

Edge focusing



Structures exist at their own scale:

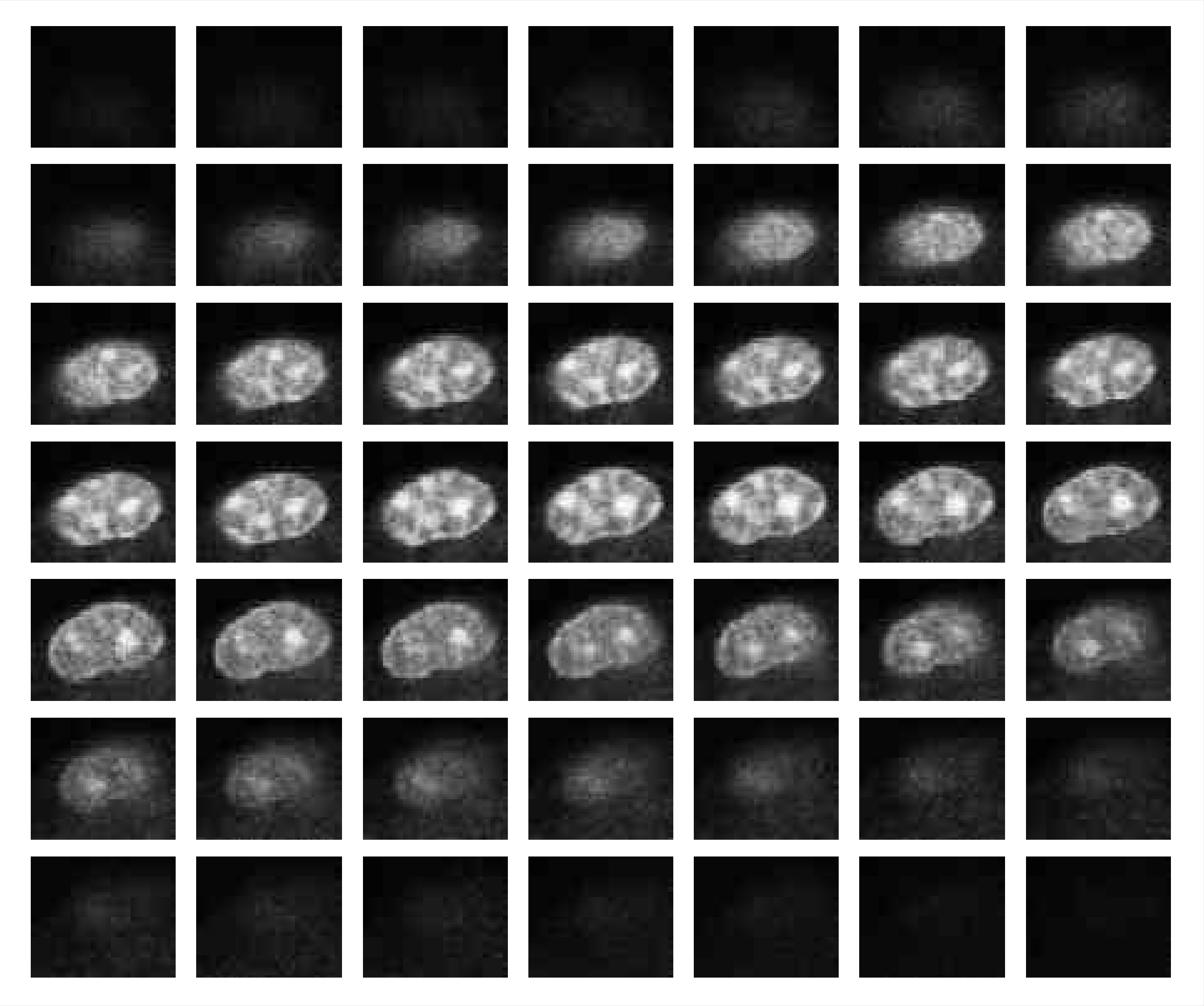


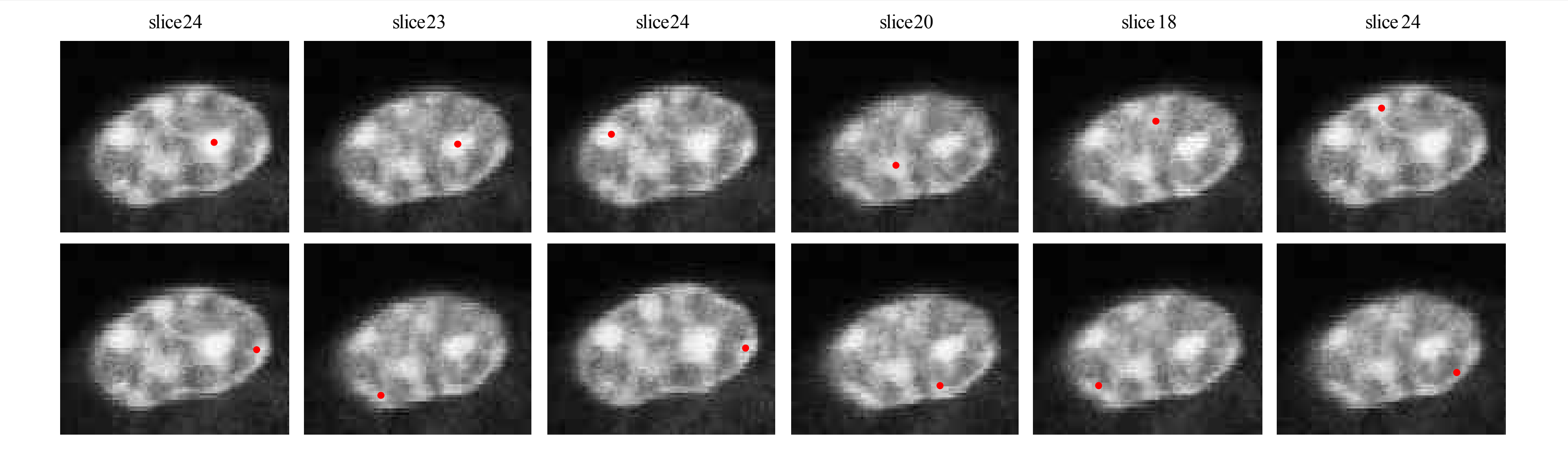
Original $\sigma = e^0$ px $\sigma = e^1$ px $\sigma = e^2$ px $\sigma = e^3$ p

Noise edges

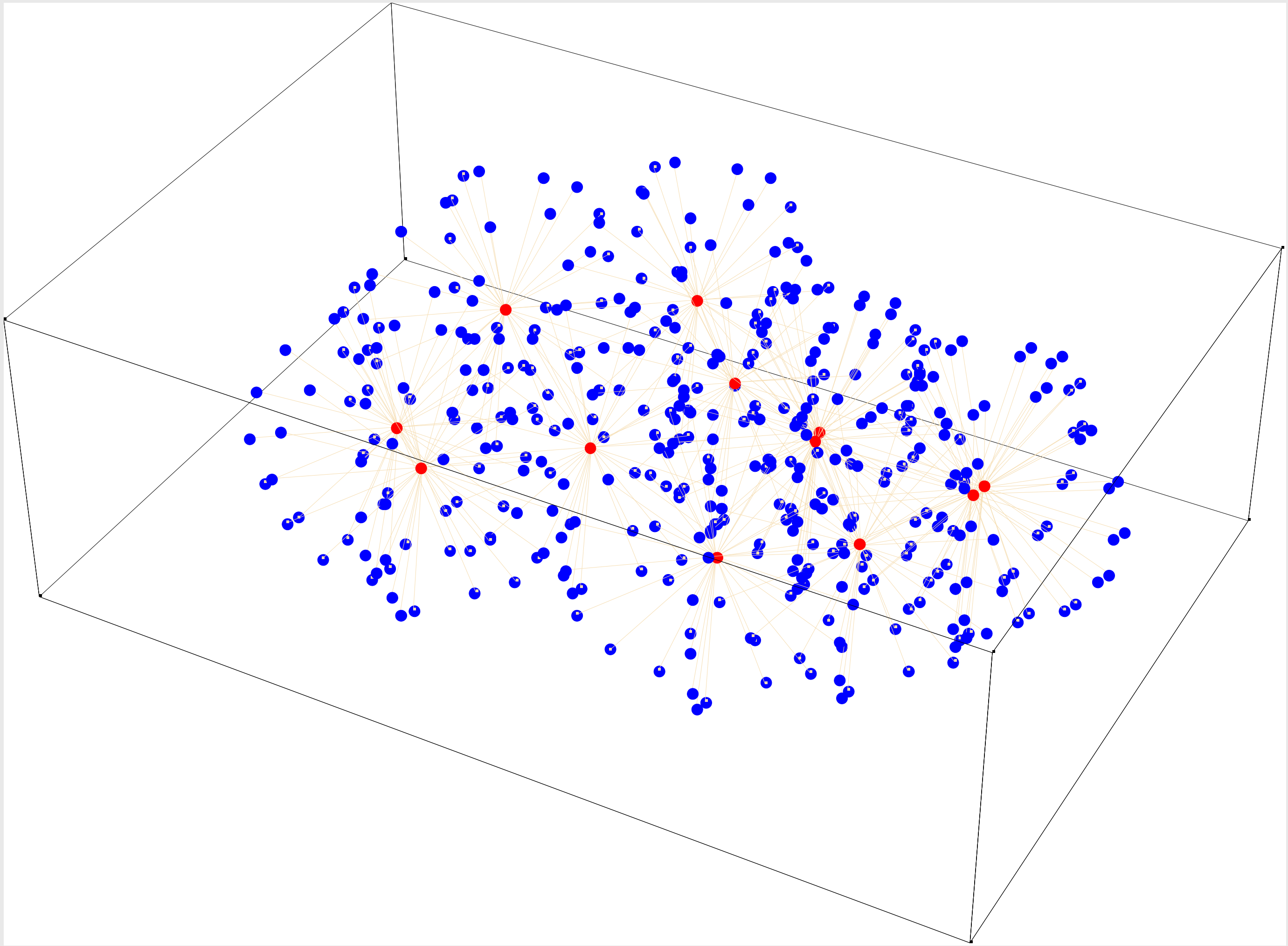
Example:

Lysosome segmentation in noisy 2-photon microscopy 3D images of macrophages.



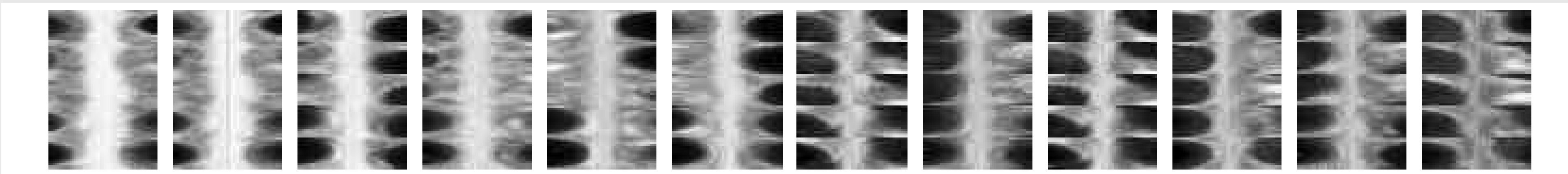


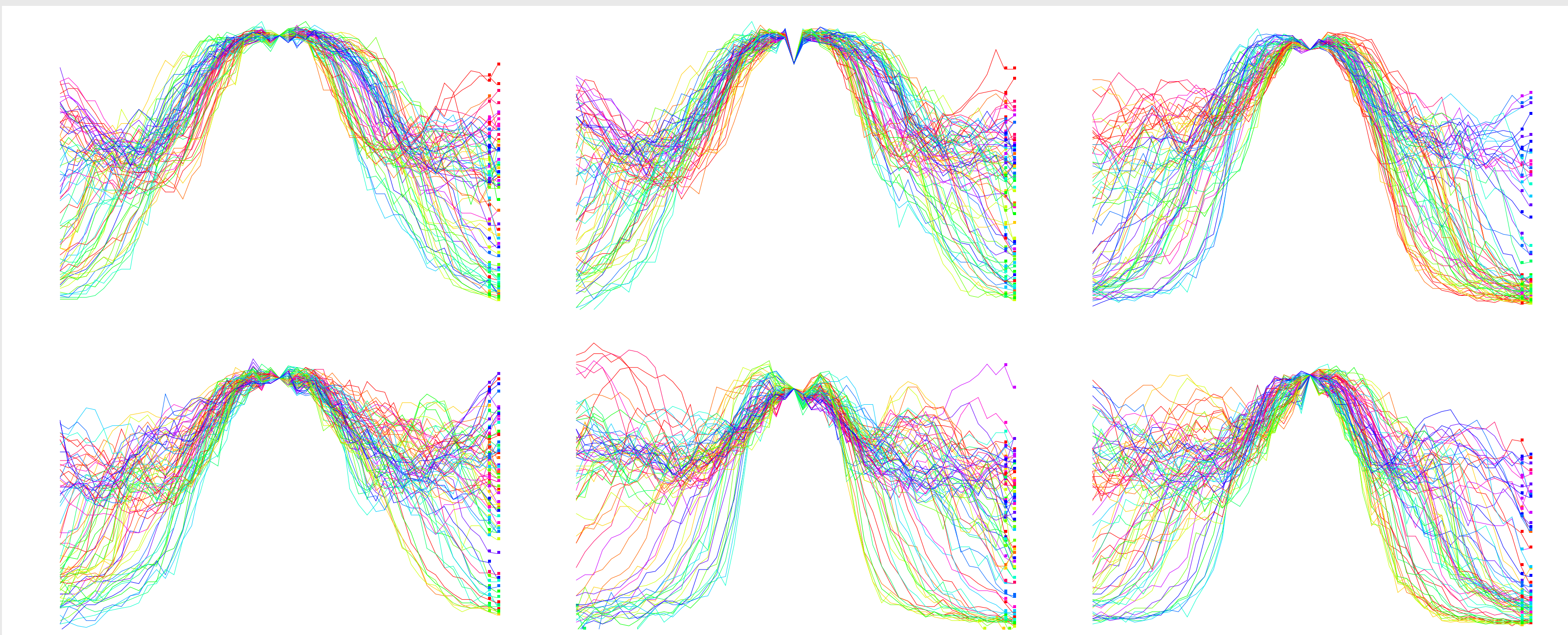
First the 3D maxima are detected at scale σ = 3 pixels



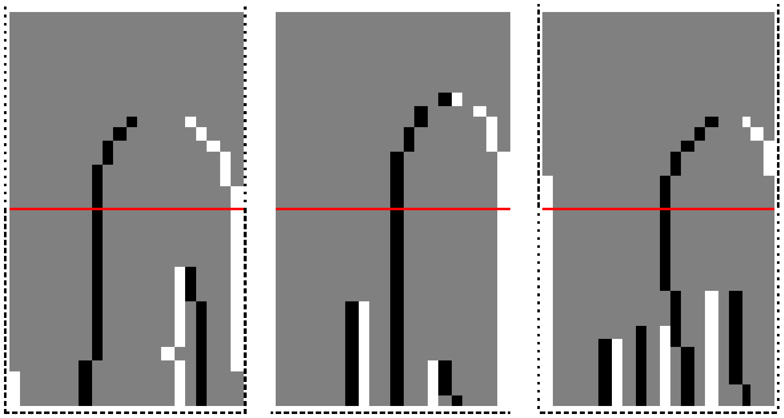
We interpolate
with cubic splines
35 radial tracks
in 35 orientations
for 12 maxima

The profiles are extremely noisy:



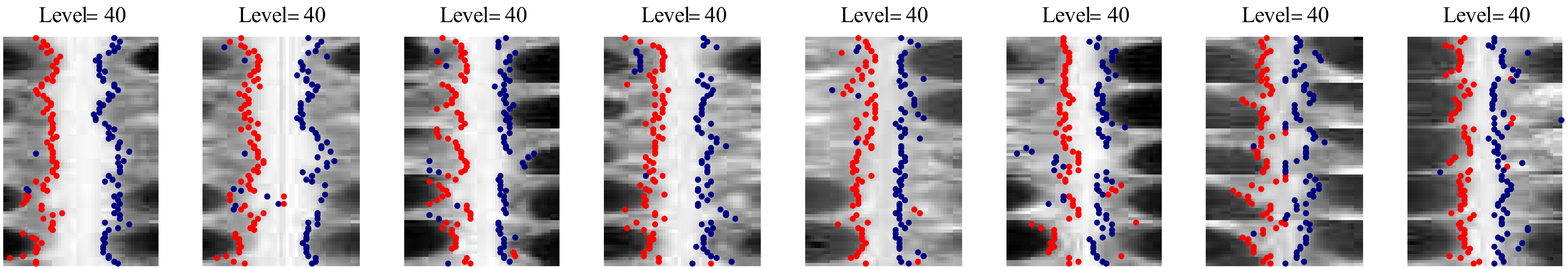


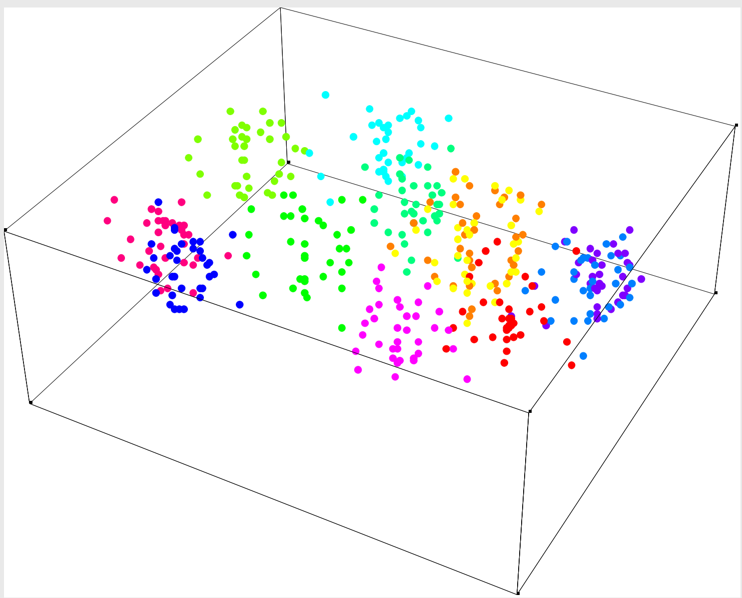
Observation: visually we can reasonably point the steepest edgepoints



Edge focusing over all profiles.

Choose a start level based on the *task*, i.e. find a single edge.





Detected 3D points per maximum.

We need a 3D shape fit function.

The 3D points are least square fit with 3D spherical harmonics:

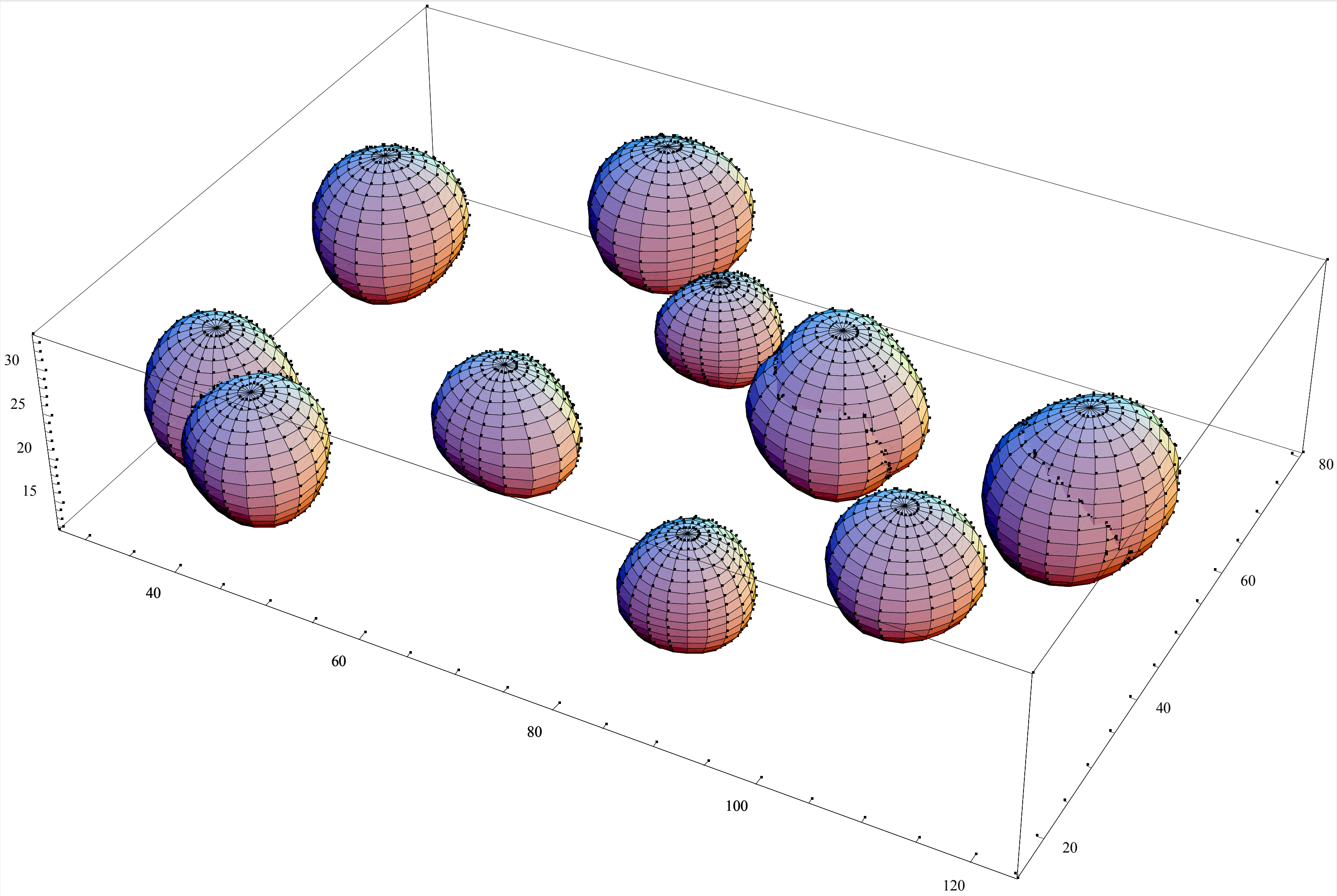
$$\left\{\frac{1}{2\sqrt{\pi}},\ \frac{1}{2}e^{-i\phi}\sqrt{\frac{3}{2\pi}}\sin(\theta),\ \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos(\theta),\right.$$

$$-\frac{1}{2}e^{i\phi}\sqrt{\frac{3}{2\pi}}\sin(\theta),\ \frac{1}{4}e^{-2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta),$$

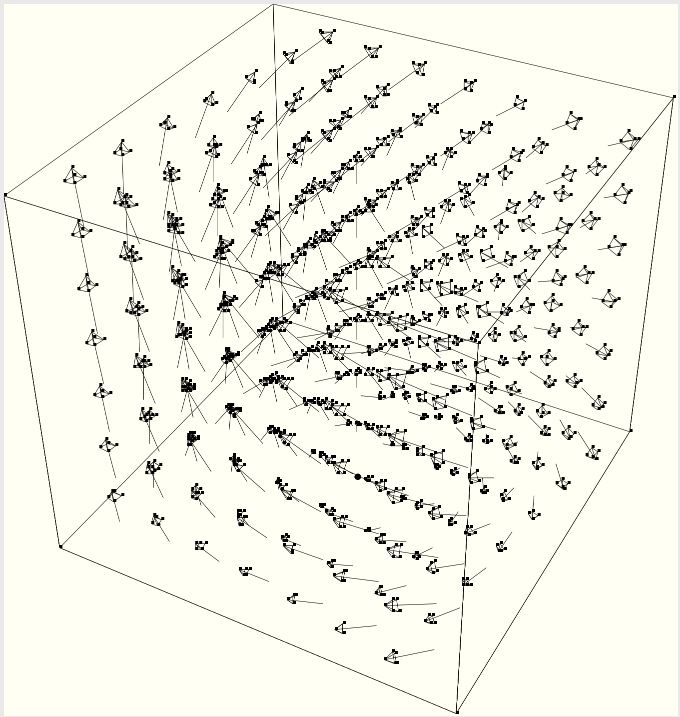
$$\frac{1}{2}e^{-i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta),\ \frac{1}{4}\sqrt{\frac{5}{\pi}}\left(3\cos^2(\theta)-1\right),$$

$$\left.-\frac{1}{2}e^{i\phi}\sqrt{\frac{15}{2\pi}}\cos(\theta)\sin(\theta),\ \frac{1}{4}e^{2i\phi}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)\right\}$$

Resulting detection:



Multi-scale optic flow



How can we find a <u>dense</u> optic flow field from a motion sequence in 2D and 3D?

Many approaches are taken:

- gradient based (or differential);
- phase-based (or frequency domain);
- correlation-based (or area);
- feature-point (or sparse data) tracking.

The Lie derivative (denoted with the symbol $\mathcal{L}_{\vec{v}}$) of a function $F(g)$ with respect to a vectorfield $\vec{v}$ is defined as $\mathcal{L}_{\vec{v}} F(g)$. The optic flow constraint equation (OFCE) states that the luminance does not change when we take the derivative along the vectorfield of the motion:

$$\mathcal{L}_{\vec{v}} F(g) \equiv 0$$

Multi-scale optic flow constraint equation:

For scalar images:

$$\mathcal{L}_{\vec{v}}\, F(g) = \vec{\vec{\nabla}}\, F.\vec{v}$$

For density images:

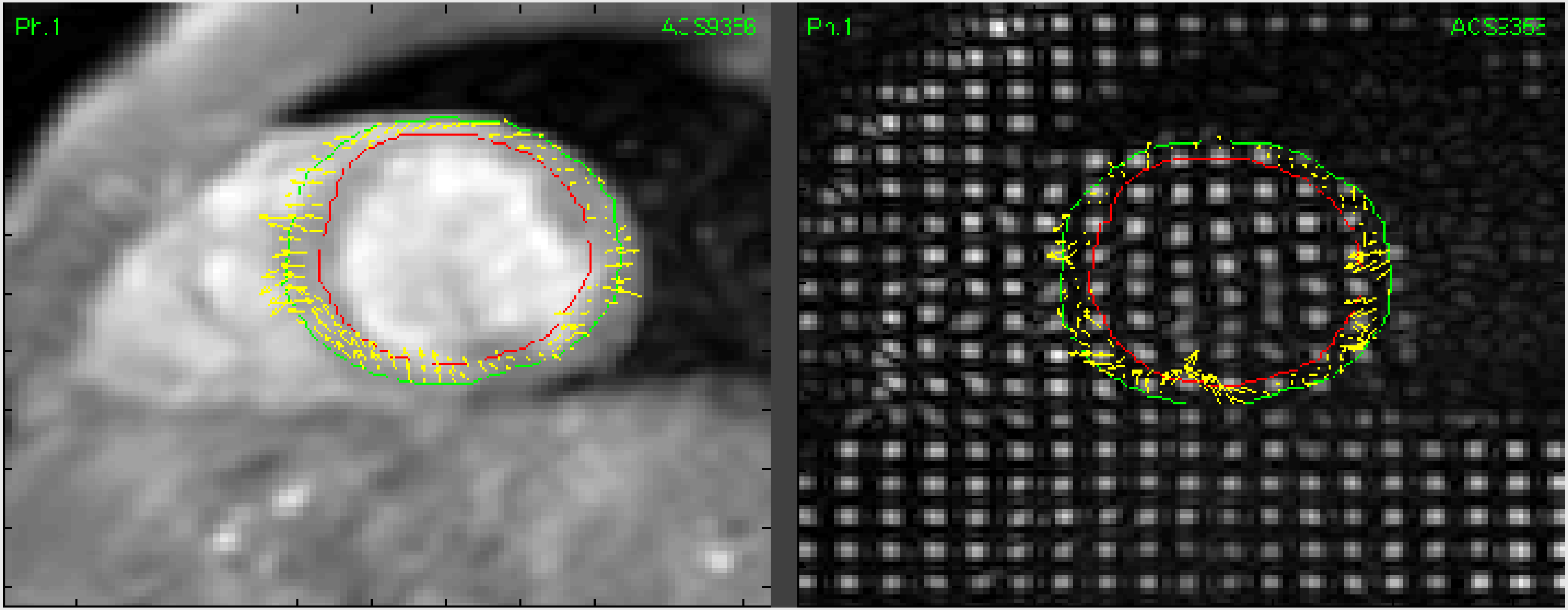
$$\mathcal{L}_{\vec{v}}\, \rho = \rho\, \mathrm{Div}\, \vec{v} + \vec{v}.\vec{\vec{\nabla}}\, \rho = 0$$

The velocity field is unknown, and this is what we want to recover from the data. We like to retrieve the velocity and its derivatives with respect to x, y, z and t.

We insert this unknown velocity field as a truncated Taylor series, truncated at first order.

Multi-scale density flow: in each pixel 8 equations of third order and 8 unknowns:

$$\begin{pmatrix}
L_x & \sigma x^2 L_{xx} & \sigma y^2 L_{xy} & \tau^2 L_{xt} & L_y & \sigma x^2 L_{xy} & \sigma y^2 L_{yy} & \tau^2 L_{yt} \\
-L_{xx} & -L_x-\sigma x^2 L_{xxx} & -\sigma y^2 L_{xxy} & -\tau^2 L_{xxt} & -L_{xy} & -\sigma x^2 L_{xxy}-L_y & -\sigma y^2 L_{xyy} & -\tau^2 L_{xyt} \\
-L_{xy} & -\sigma x^2 L_{xxy} & -L_x-\sigma y^2 L_{xyy} & -\tau^2 L_{xyt} & -L_{yy} & -\sigma x^2 L_{xyy} & -L_y-\sigma y^2 L_{yyy} & -\tau^2 L_{yyt} \\
-L_{xt} & -\sigma x^2 L_{xxt} & -\sigma y^2 L_{xyt} & -L_x-\tau^2 L_{xtt} & -L_{yt} & -\sigma x^2 L_{xyt} & -\sigma y^2 L_{yyt} & -L_y-\tau^2 L_{ytt} \\
L_y & \sigma x^2 L_{xy} & \sigma y^2 L_{yy} & \tau^2 L_{yt} & -L_x & -\sigma x^2 L_{xx} & -\sigma y^2 L_{xy} & -\tau^2 L_{xt} \\
-L_{xy} & -\sigma x^2 L_{xxy}-L_y & -\sigma y^2 L_{xyy} & -\tau^2 L_{xyt} & L_{xx} & L_x+\sigma x^2 L_{xxx} & \sigma y^2 L_{xxy} & \tau^2 L_{xxt} \\
-L_{yy} & -\sigma x^2 L_{xyy} & -L_y-\sigma y^2 L_{yyy} & -\tau^2 L_{yyt} & L_{xy} & \sigma x^2 L_{xxy} & L_x+\sigma y^2 L_{xyy} & \tau^2 L_{xyt} \\
-L_{yt} & -\sigma x^2 L_{xyt} & -\sigma y^2 L_{yyt} & -L_y-\tau^2 L_{ytt} & L_{xt} & \sigma x^2 L_{xxt} & \sigma y^2 L_{xyt} & L_x+\tau^2 L_{xtt}
\end{pmatrix}
\begin{pmatrix} u \\ u_x \\ u_y \\ u_t \\ v \\ v_x \\ v_y \\ v_t \end{pmatrix}
=
\begin{pmatrix} -L_t \\ L_{xt} \\ L_{yt} \\ L_{tt} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$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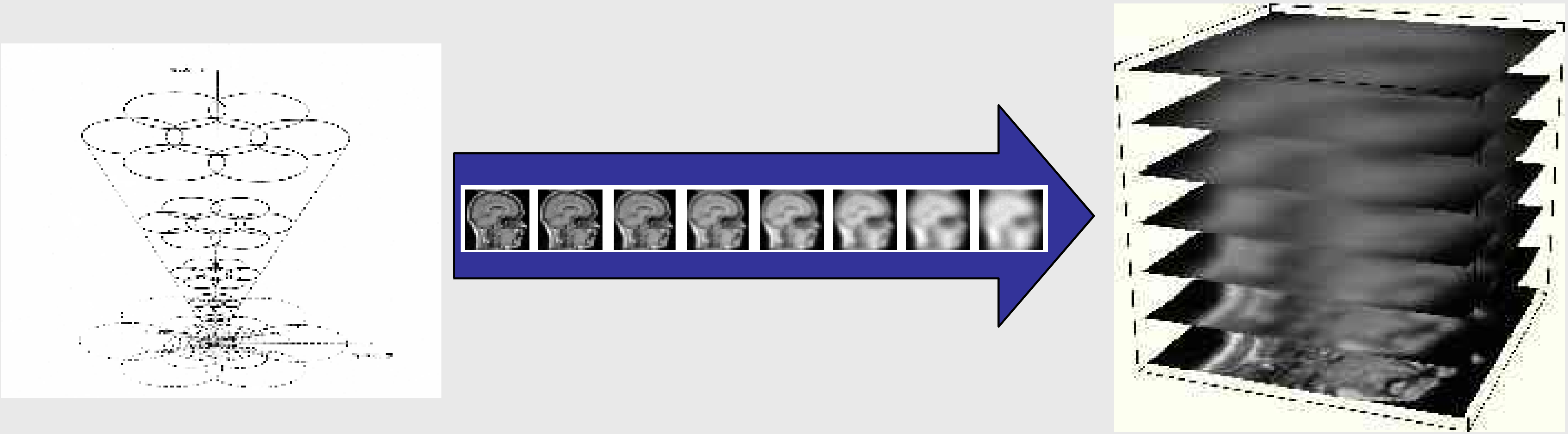
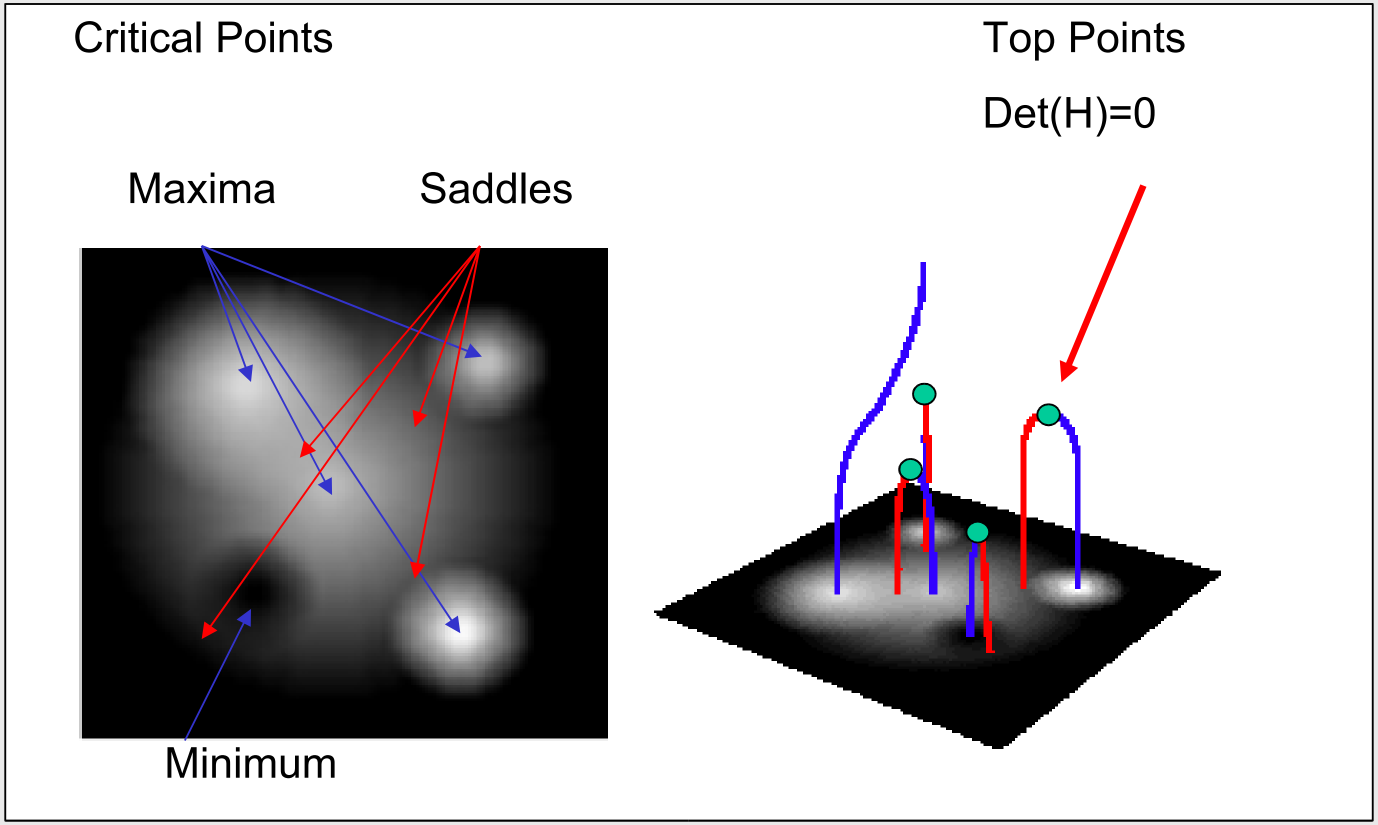
For *hierarchical image reasoning* we consider images on many resolutions simultaneously (like the eye)

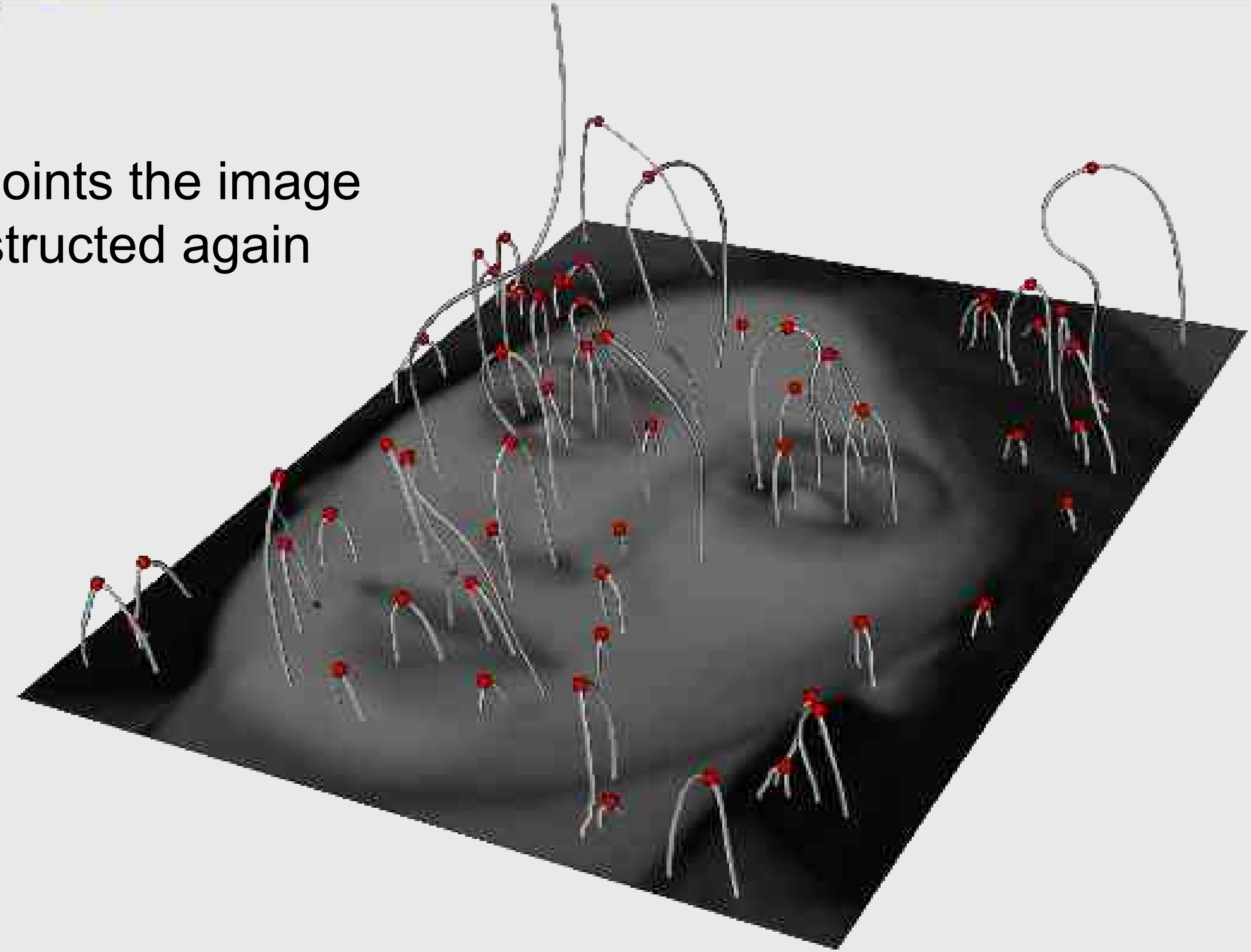


scale-space

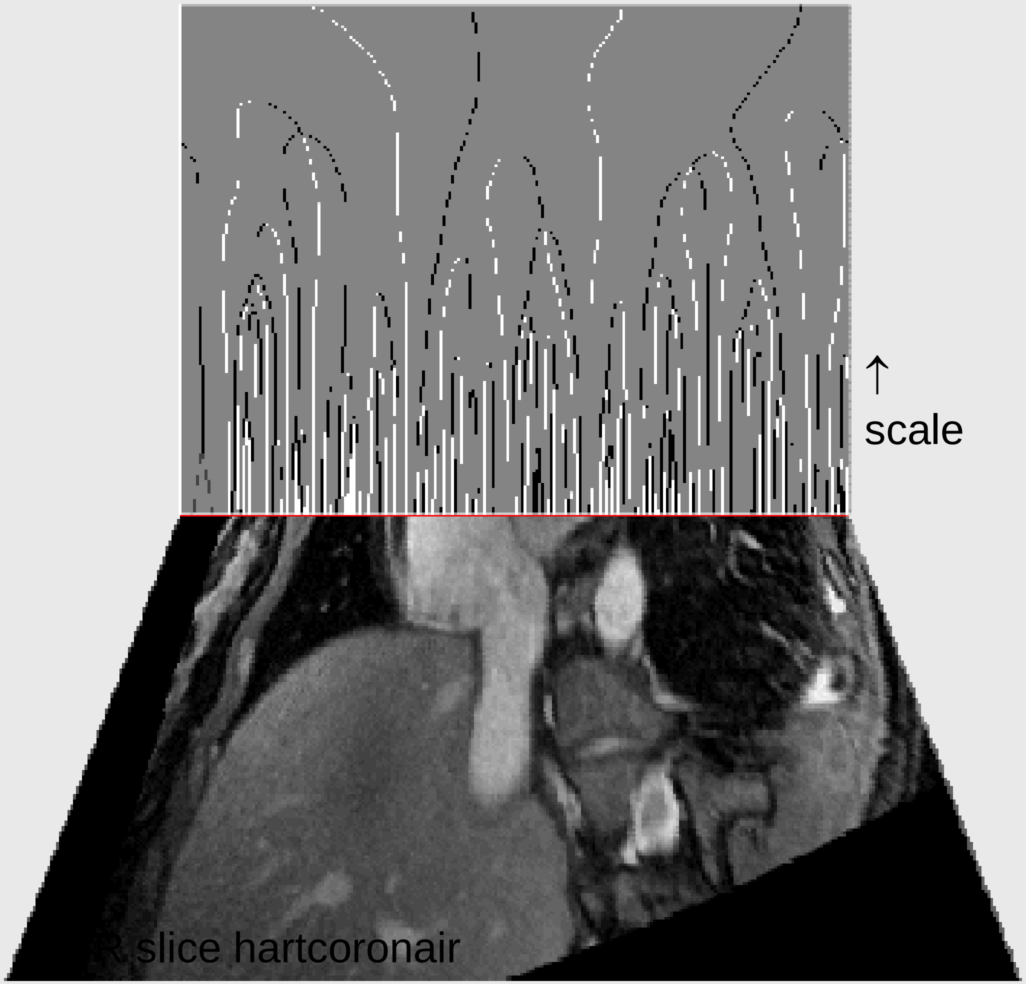
Critical Points, Paths and Top Points

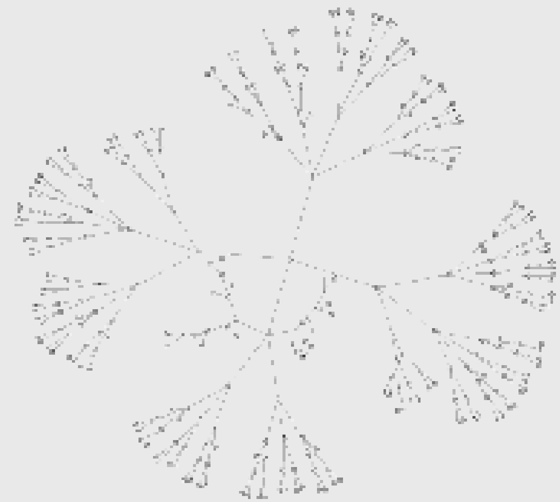


From the toppoints the image can be reconstructed again

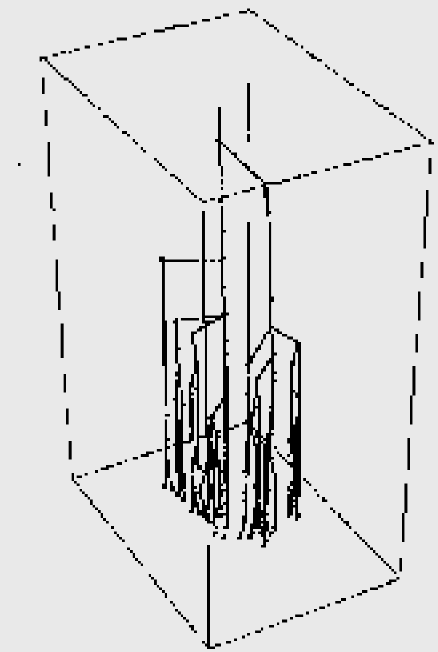


• toppoints





• graph theory



A new paradigm in multi-scale computer vision:

Hierarchical reasoning by graphs

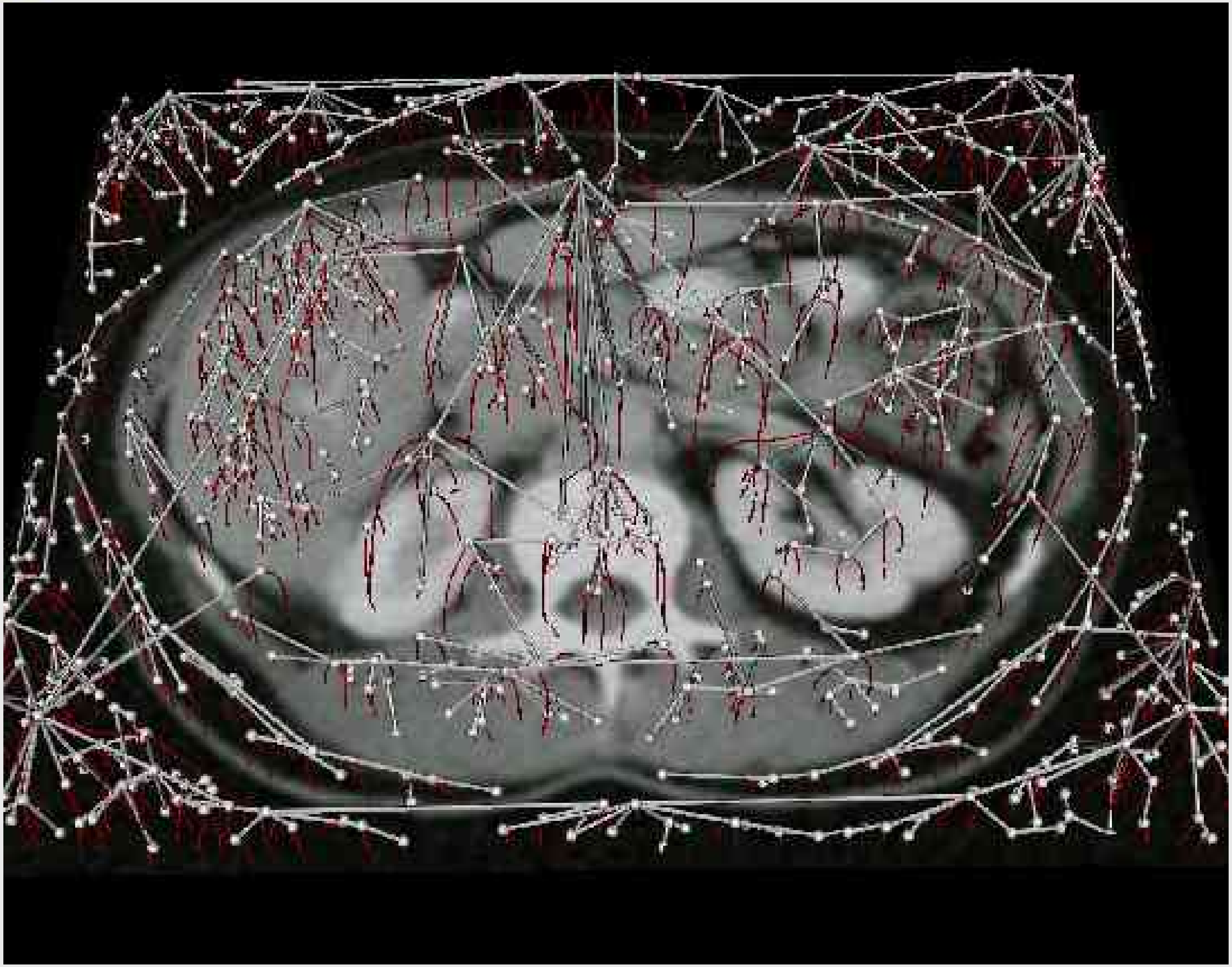
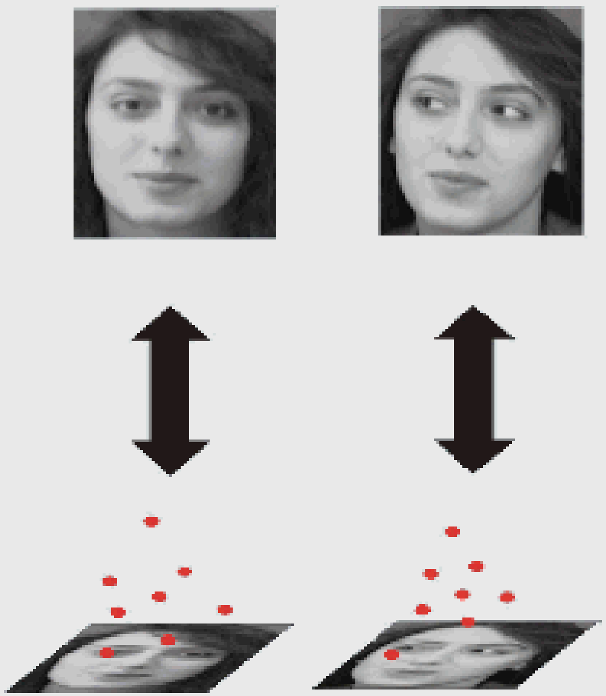
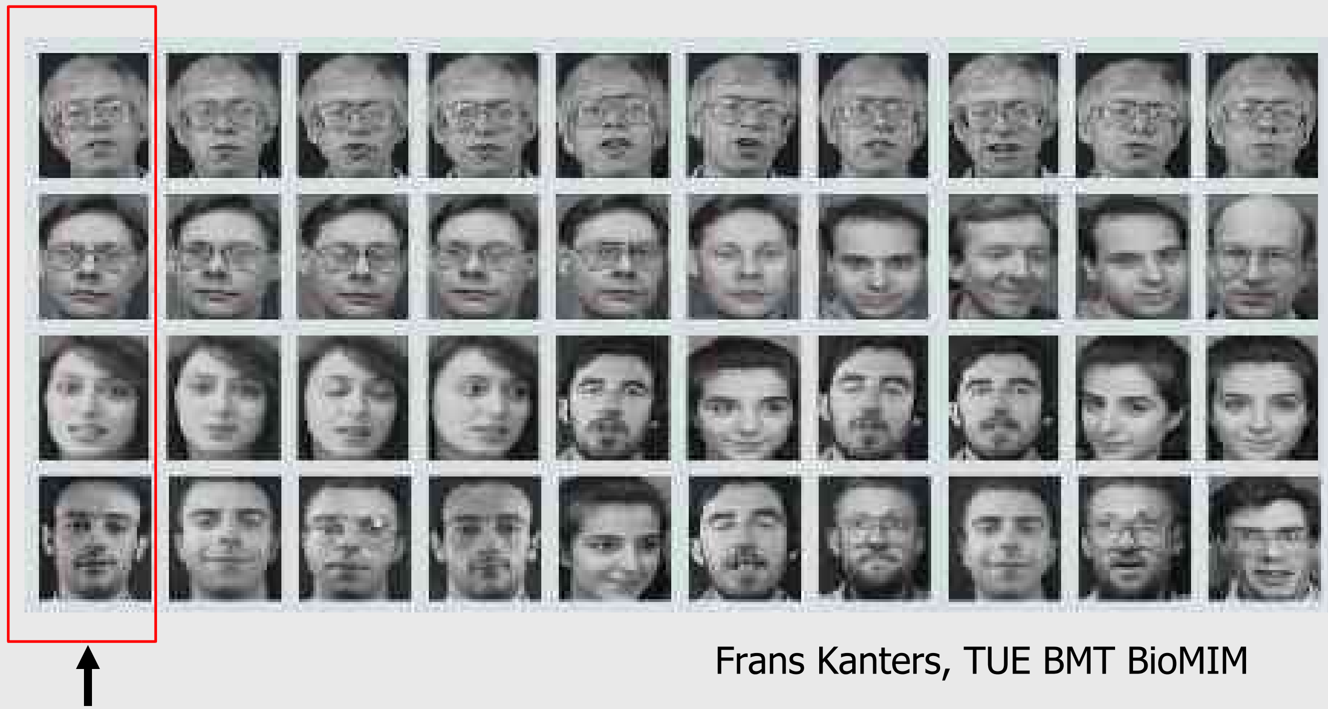


Image guided database retrieval





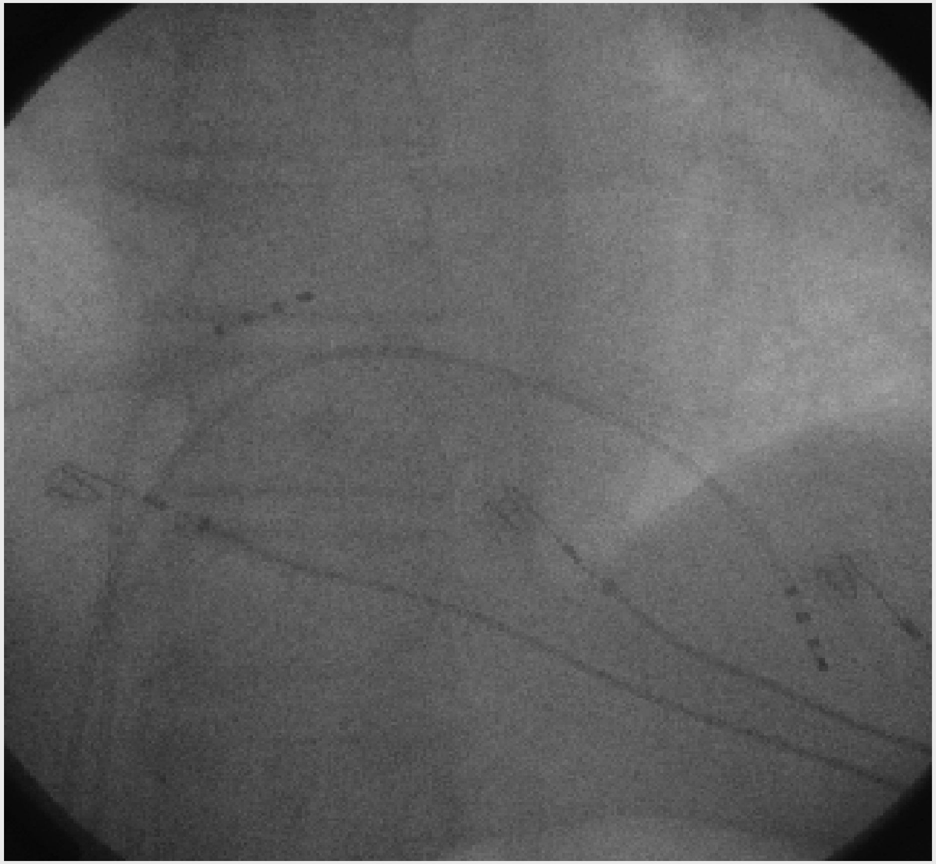
Task

Frans Kanters, TUE BMT BioMIM

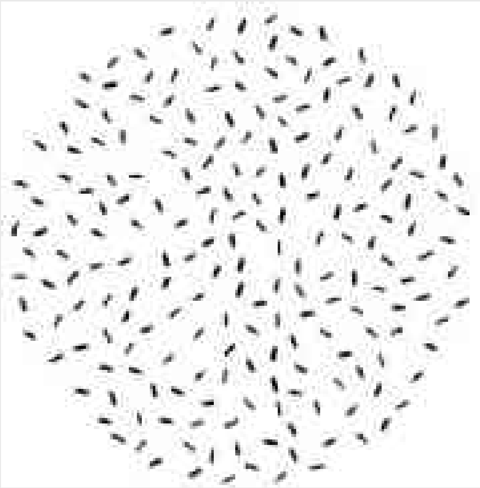
Point cloud matching (earth mover distance), very efficient

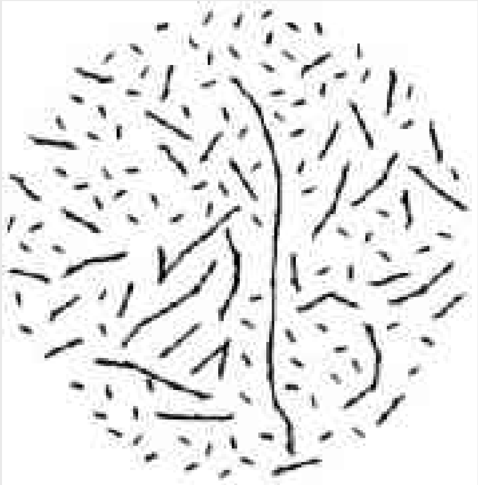
We can now reconstruct from the toppoints the image again (with Mathematica)

Catheter & electrode detection



Perceptual grouping (Gestalt) from orientations: robust detection

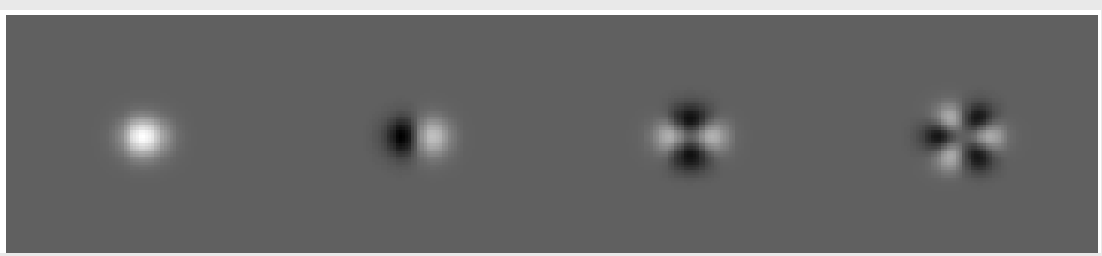




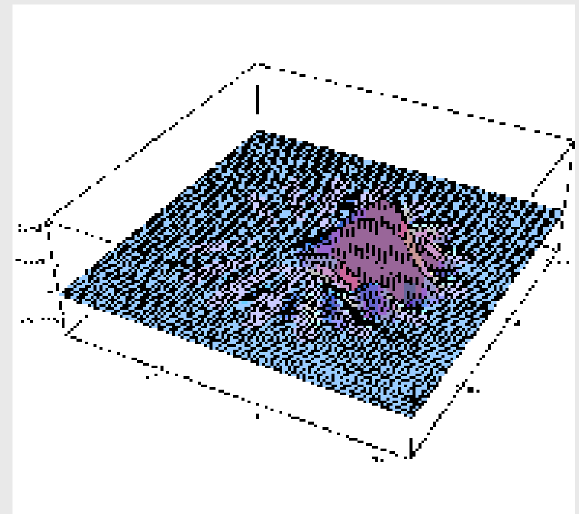
Context filters

$$\Phi_n(z,\sigma) = a_n(-\sigma\partial)^n e^{-\frac{z\bar{z}}{\sigma^2}},$$

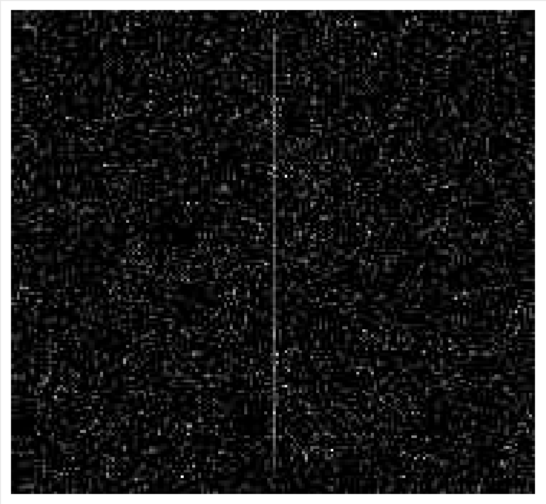
$$\Phi_{-n}(z,\sigma) = \overline{a_n}(-\sigma\,\bar{\partial})^n e^{-\frac{z\bar{z}}{\sigma^2}};n \geq 0$$

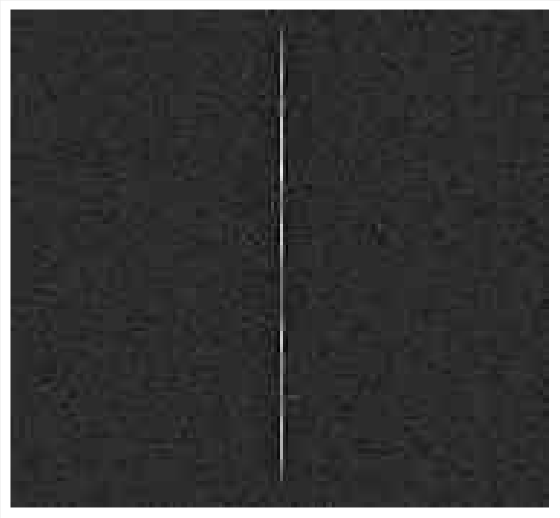


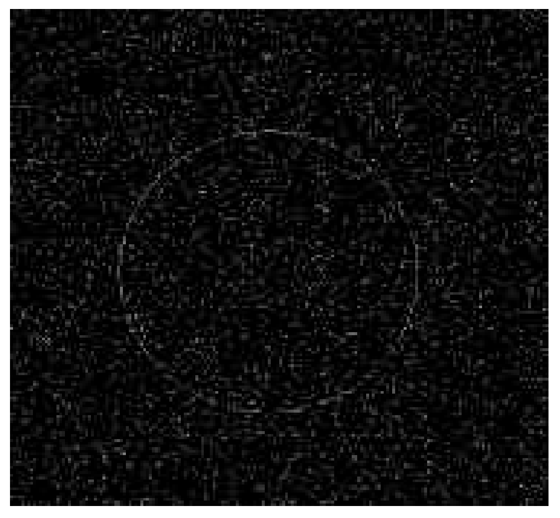
Gaussian Orientation Bundle

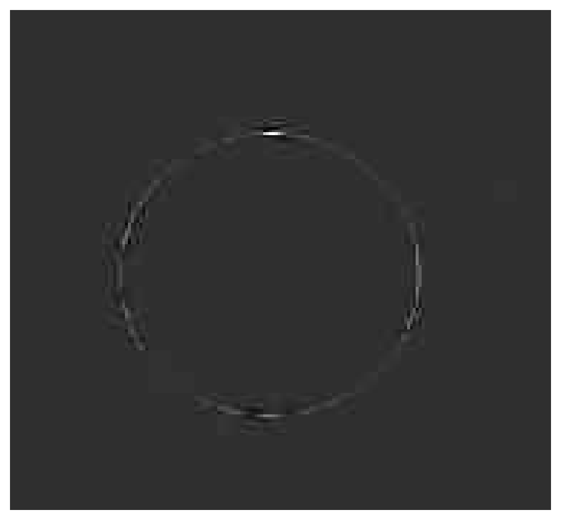


Strong non-linear filtering in orientation space

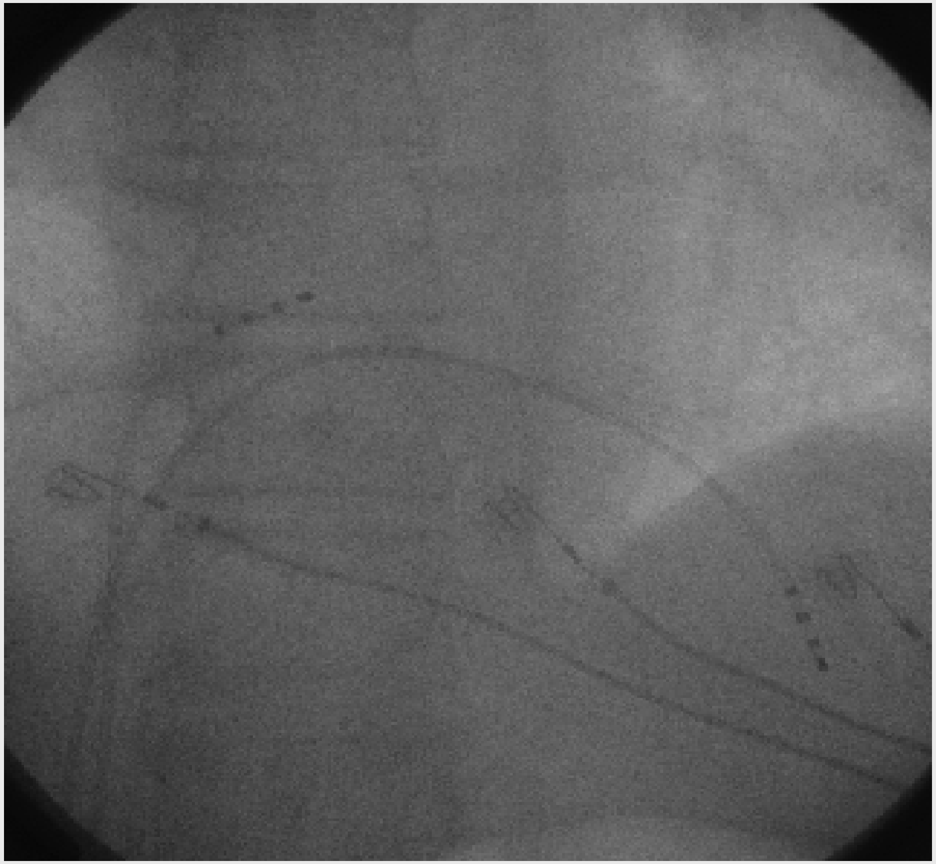


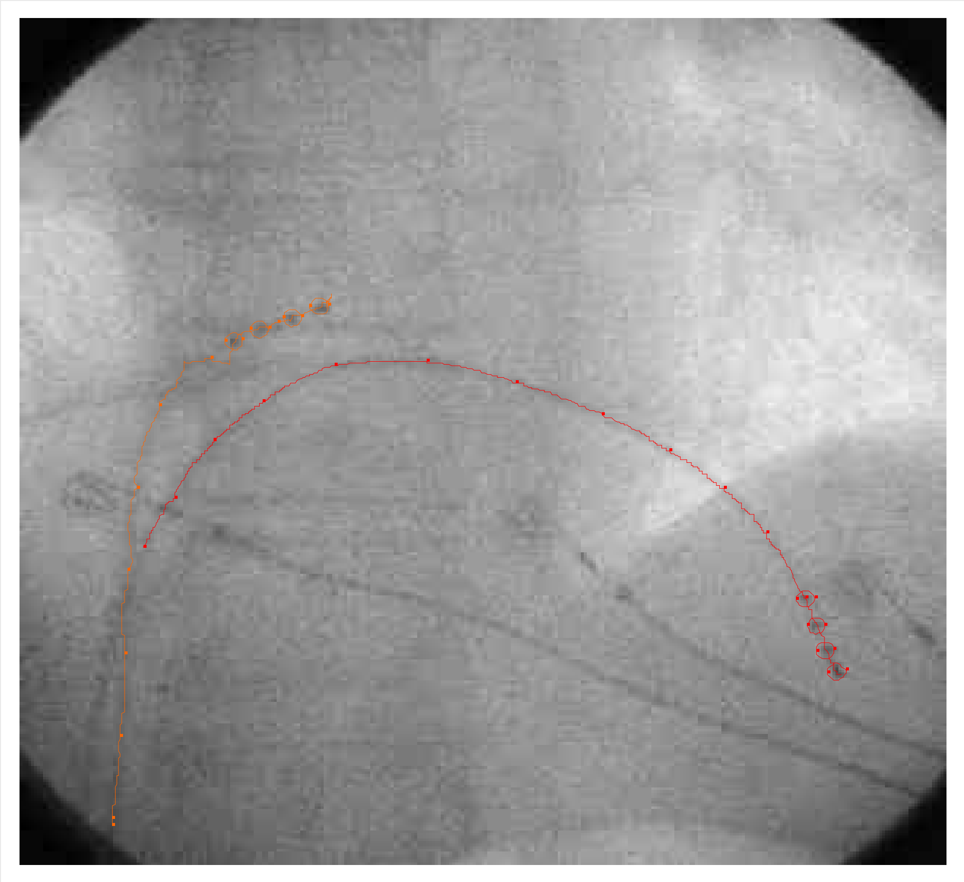




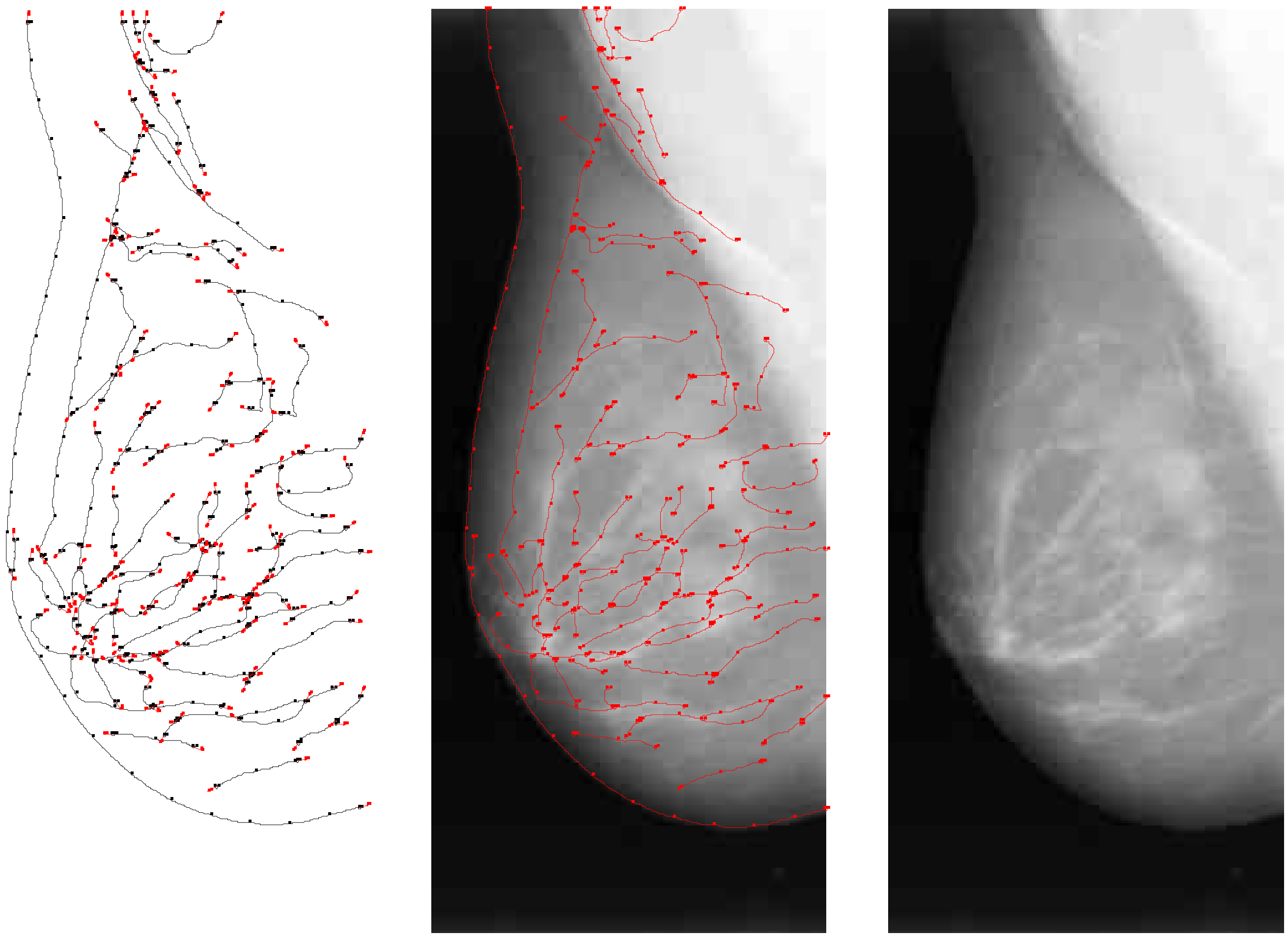


Context orientation bundle with tensor voting





Vessel detection for Computer Aided Diagnosis in Mammography



E. Franken, M. van Almsick

MathVisionTools

List of currently available functions:

- Differential Geometry
  - Gaussian derivatives
  - for any order
  - for N dimensions
- Import / Export
  - any dialect DICOM
  - high field MRI
  - 3D ultrasound
  - 2-photon microscopy
- Orientation analysis
  - Polar Fourier Transform
  - 2D Hankel Transform

Soon available:

- MIP (perspective and orthogonal)
- Image registration in 2D and 3D
- Mutual entropy & correlation measure
- Lung nodule detection
- Catheter detection by 3D orientation bundle
- Tensor voting for perceptual grouping
- Dynamic shape Eigenmode analysis
- Automatic updating system

Scheduled:

- Nonlinear image registration
- PDE based edge preserving smoothing
- Motion from dense optic flow fields
- Active contours atlas mapping
- Snakes and levelsets
- Image retrieval (multi-scale)
- Multi-scale texture classification

The numerical *and* symbolic power of *Mathematica* is used

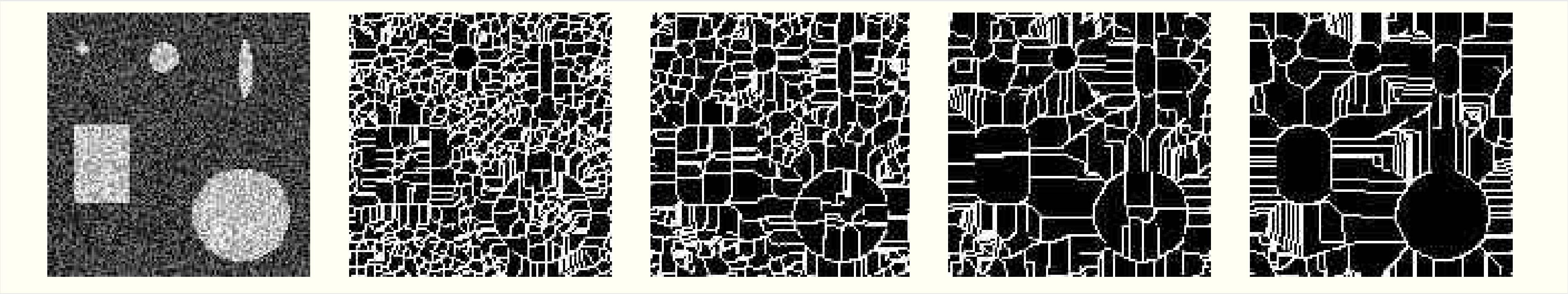
You are invited for a mutual collaboration to develop the library (on exchange basis)

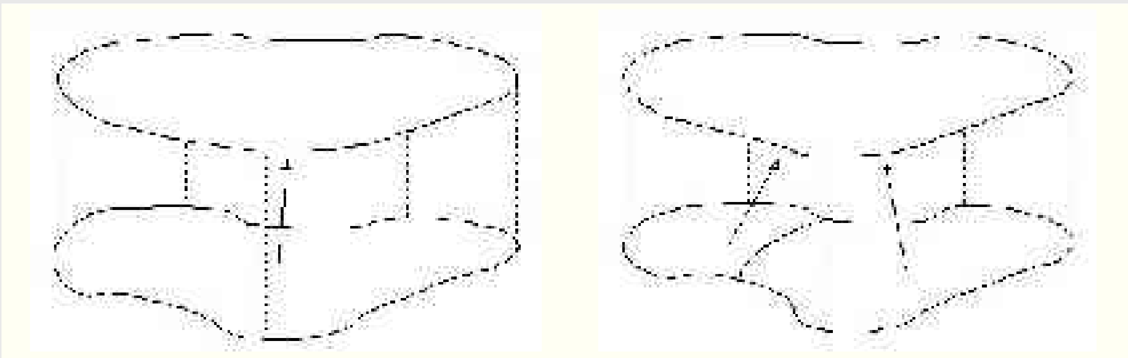
Contact: prof. Bart M. ter Haar Romeny, PhD
dipl.ing. Markus van Almsick

Multi-scale watershed segmentation

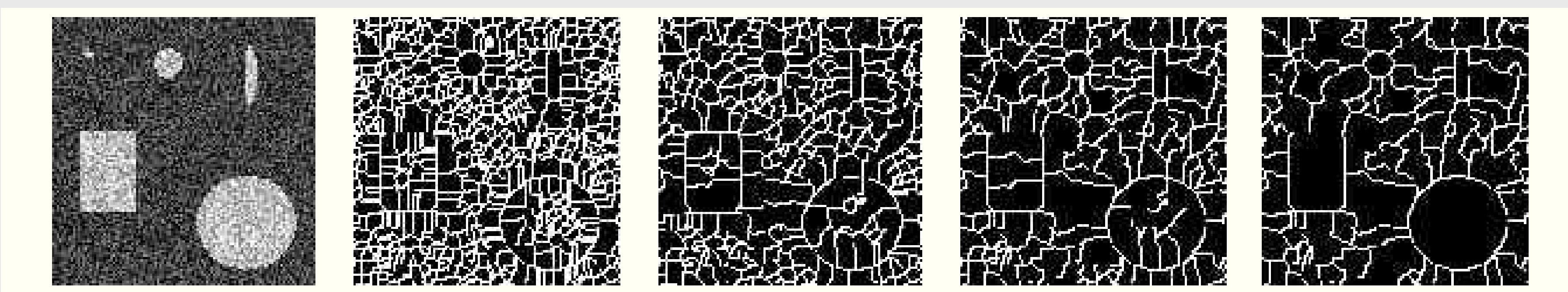
Watershed are the boundaries of merging water basins, when the image landscape is immersed by punching the minima.

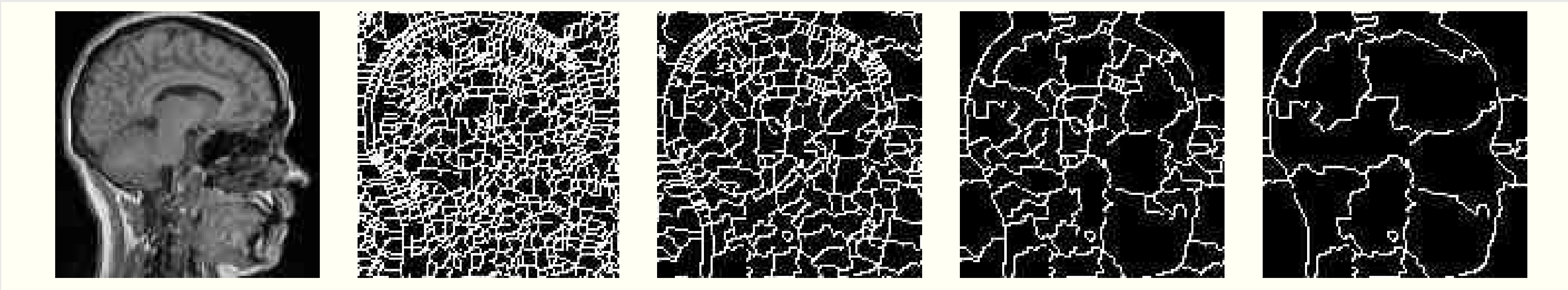
At larger scale the boundaries get blurred, rounded and dislocated.



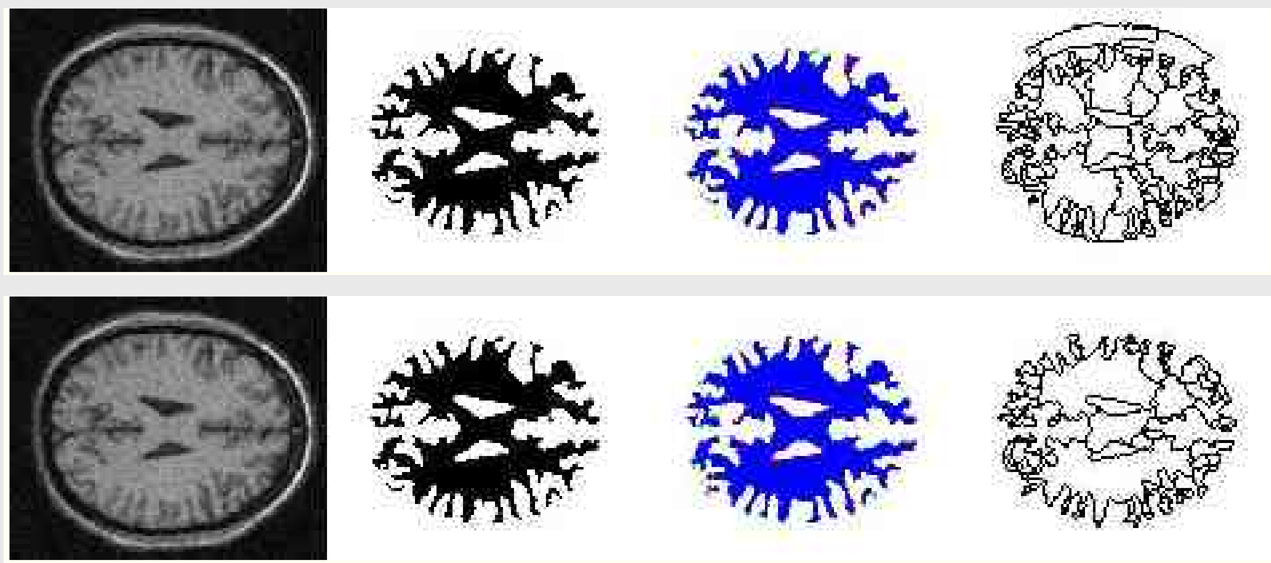


Regions of different scales can be linked by calculating the largest overlap with the region in the scales just above.



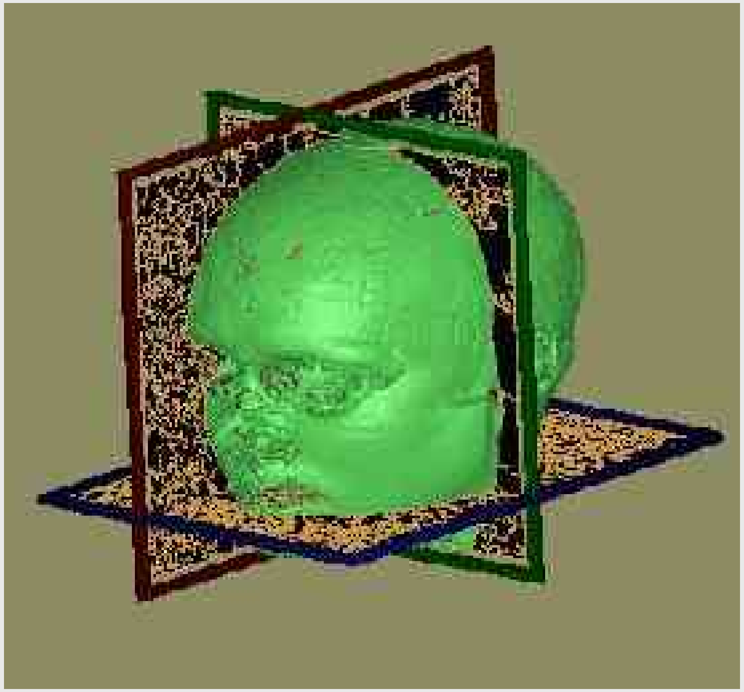


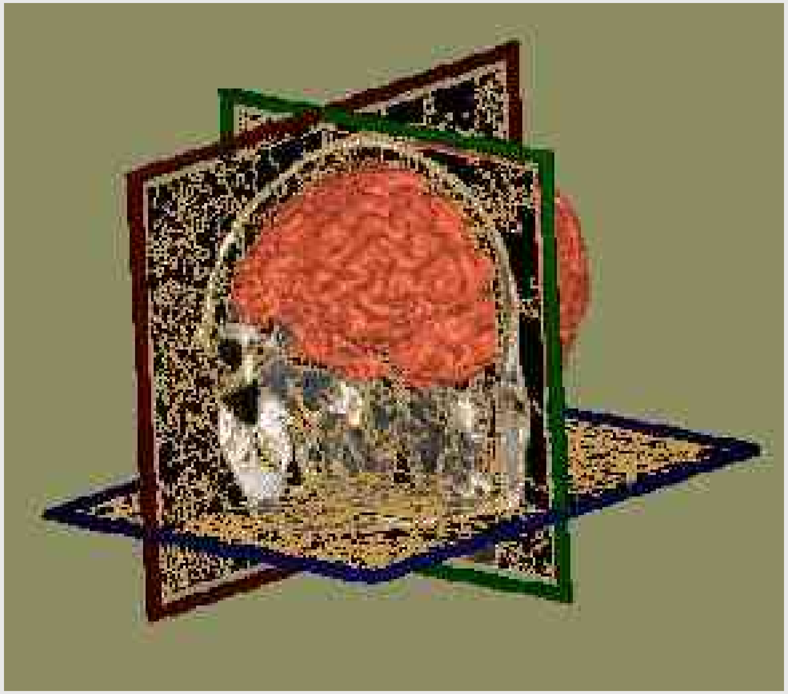
The method is often combined with nonlinear diffusion schemes



E. Dam, ITU

Nabla Vision is an interactive 3D watershed segmentation tool developed by the University of Copenhagen.

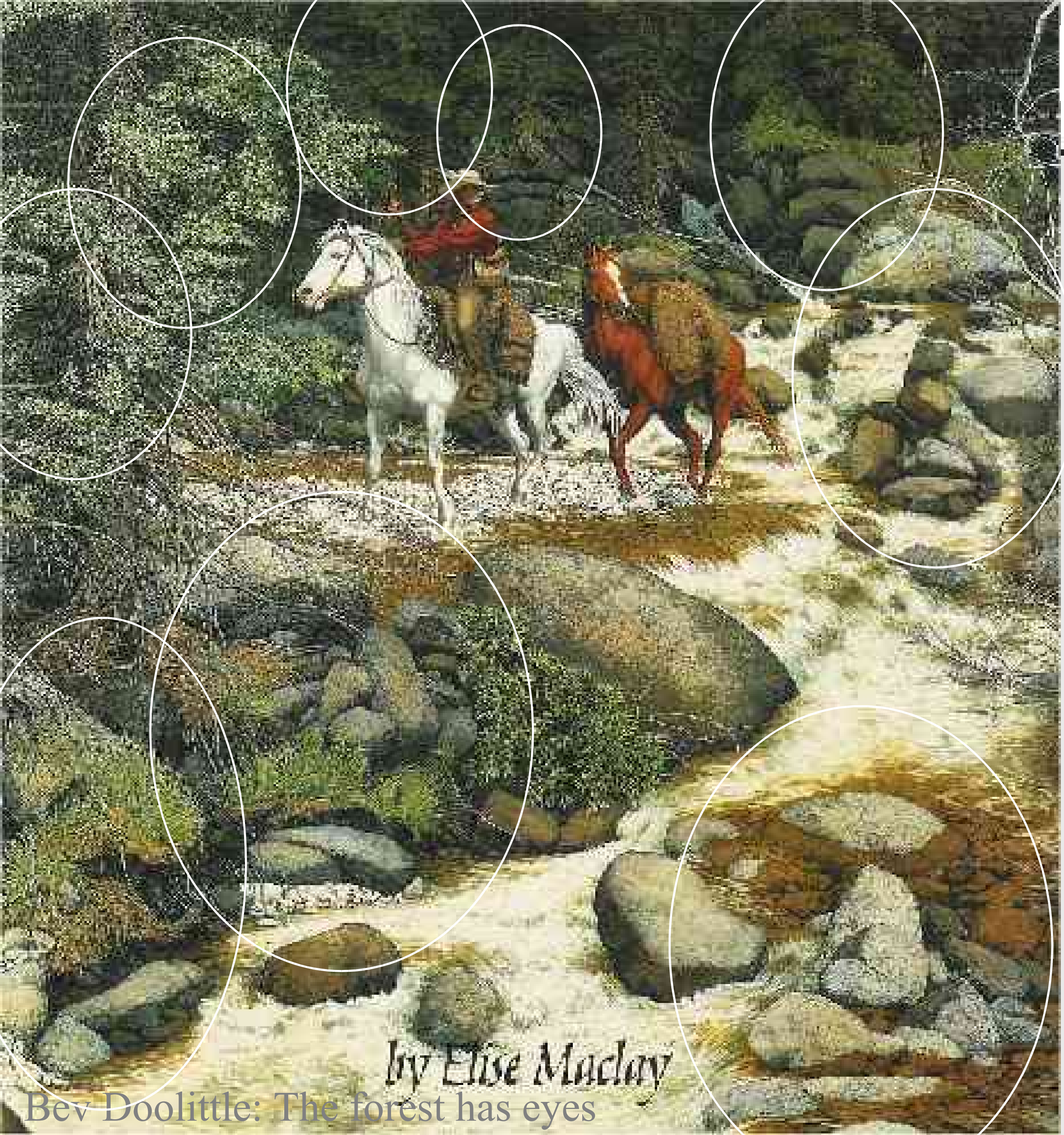




Sculpture the 3D shape by successively clicking precalculated finer scale watershed details.

Color invariant: $\varepsilon = \frac{1}{e}\frac{\partial e}{\partial \lambda}$

Yellow-blue edges: $\sqrt{\frac{(e\,e_{xz} - e_x\,e)^2 + (e\,e_{yz} - e_y\,e_z)^2}{e^4}}$



The challenge

How do we do it?